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Knot Theory and Quantum Information

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Musings on SVD and pseudo entanglement entropies **JHEP11(2024)103**

Pick a quantum state $|\varphi\rangle$ in Hilbert space $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$ to obtain the density matrix

$$\tau^{\varphi|\varphi} = \frac{|\varphi\rangle\langle\varphi|}{\langle\varphi|\varphi\rangle} \tag{1}$$

and reduced density matrix for A by tracing over the complement B : $\tau_A^{\varphi|\varphi} = \mathrm{Tr}_B(\tau^{\varphi|\varphi})$

The **Entanglement Entropy** is defined as:

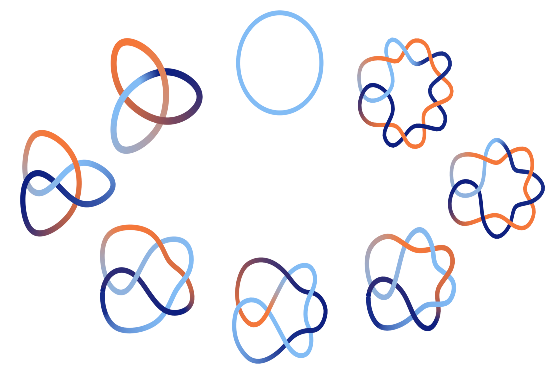
$$S_E = -\mathrm{Tr}\,(\tau_A^{\varphi|\varphi} \log \tau_A^{\varphi|\varphi}) = -\sum_i \lambda_i \log \lambda_i \tag{2}$$

For two such states $|\varphi\rangle$ and $|\psi\rangle$, we define a transition matrix

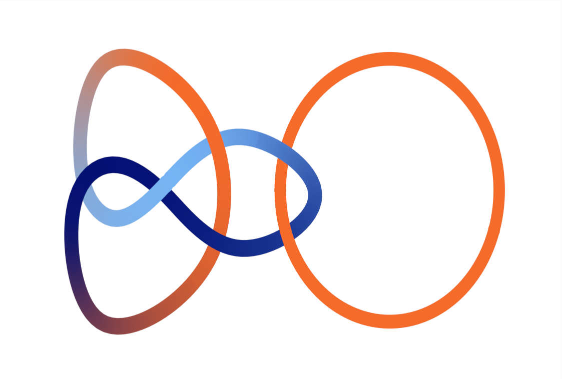
$$\tau^{\varphi|\psi} = \frac{|\varphi\rangle\langle\psi|}{\langle\psi|\varphi\rangle} \tag{3}$$

and analogous reduced transition matrix for A : $\tau_A^{\varphi|\psi} = \mathrm{Tr}_B(\tau^{\varphi|\psi})$

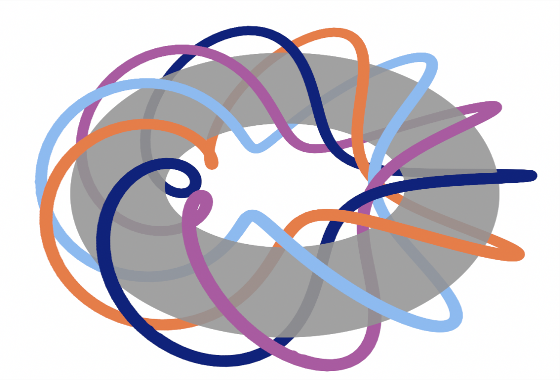
- **Pseudo Entanglement Entropy** S_P: consider the **eigenvalues** of $\tau_A^{\varphi|\psi}$ in (2)
- **SVD Entanglement Entropy** S_{SVD}: consider the **singular values** of $\tau_A^{\varphi|\psi}$ in (2)



Twist knots $\mathcal{K}_p$



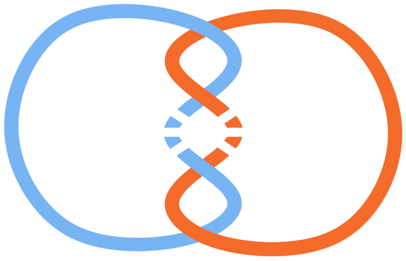
Link of type $\mathcal{K}_p\#2_1^2$



Torus link $T(P,Q)$

$$|\mathcal{L}^n\rangle = \sum_{j_1,j_2,\cdots,j_n} C^{\mathcal{L}^n}_{j_1 j_2 \cdots j_n} |j_1\rangle \otimes |j_2\rangle \otimes \cdots \otimes |j_n\rangle \tag{4}$$

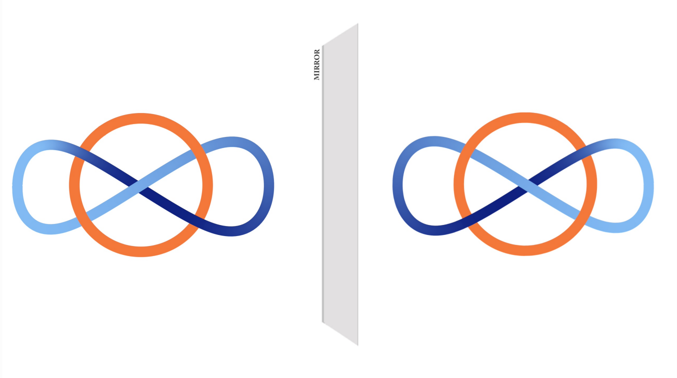
Pseudo-metric in U(1)



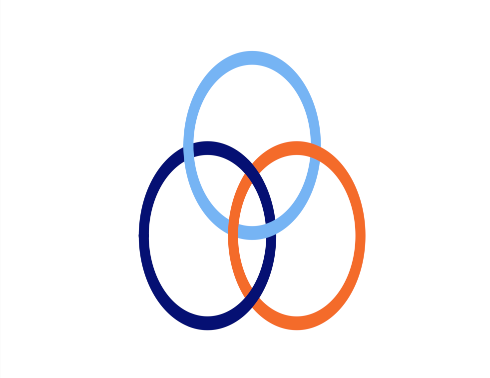
$$|\Delta S_{\mathrm{SVD}}| = \frac{1}{2}\log\left(\frac{(\gcd(k, l_1 l_2))^2}{\gcd(k, l_1)\cdot\gcd(k, l_2)}\right) \quad (5)$$

Chirality in general (compact) gauge groups

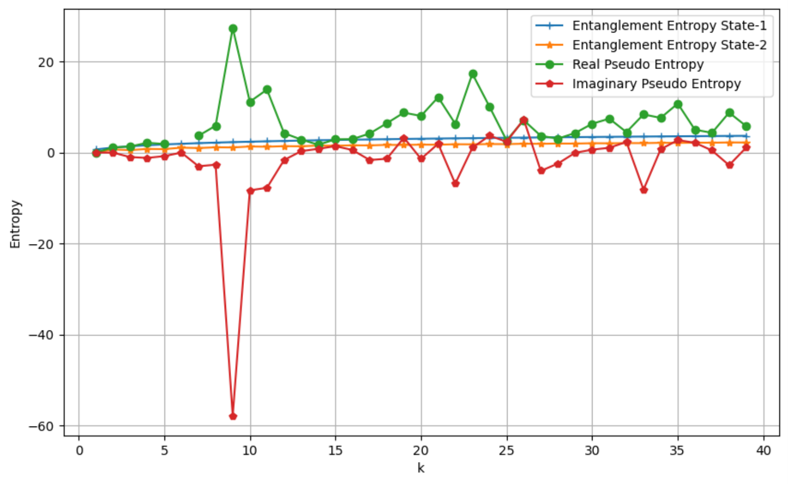
A knot $\mathcal{K}$ is chiral if it not is isotopic to its mirror image $\mathcal{K}^{\star}$; amphichiral otherwise



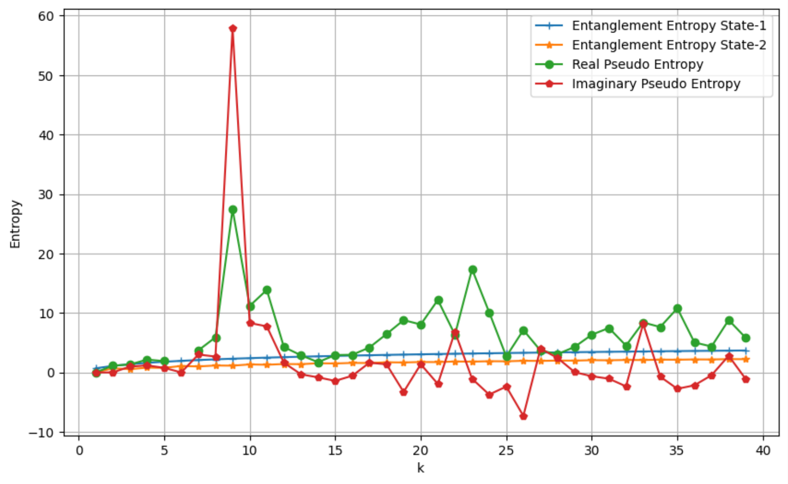
Whitehead link(s)



Borromean link



$|\mathcal{H}\rangle\langle\mathcal{W}|$



$|\mathcal{H}^{\star}\rangle\langle\mathcal{W}^{\star}|$

$$|\mathcal{L}\rangle \longrightarrow |\mathcal{L}^{\star}\rangle \implies C^{\mathcal{L}}_{mn}(q) \longrightarrow C^{\mathcal{L}^{\star}}_{mn}(q) = C^{\mathcal{L}}_{mn}(q^{-1}) = C^{\mathcal{L}}_{mn}(\overline{q}) = \overline{C^{\mathcal{L}}_{mn}(q)}$$

$$\implies \lambda \longrightarrow \overline{\lambda} \implies S_{\mathrm{P}} \longrightarrow \overline{S_{\mathrm{P}}} \implies S_{\mathrm{P}}^{\varphi^{\star}|\psi^{\star}} = S_{\mathrm{P}}^{\psi|\varphi}$$

References

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