Does time always slow down as gravity increases?

Some remarks on gravitational time dilation

Andrzej Okołów

December 5, 2025

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- Is this notion correct?
- Is the time dilation ratio correlated with the spacetime curvature?

Imprecise statements

♠ The statement *time slows down as gravity increases* and similar are imprecise descriptions of gravitational time dilation because:

- ♣ it is not clear what the term “time” means here;
- ♣ it is not clear what the verb “slows down” means here;
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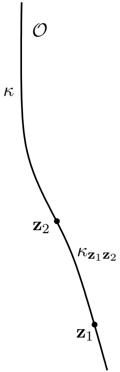
♠ Statements like *time slows down* may be a source of a misconception that passage of time can be characterized by a rate/speed and that time dilation occurs if this rate/speed is different at different locations.

Definition

♠ Gravitational time dilation concerns proper time.



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♠ The lapse $\Delta\tau$ of proper time is the 'spacetime length' of the interval $\kappa_{\mathbf{z}_1\mathbf{z}_2}$:

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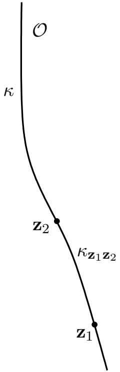

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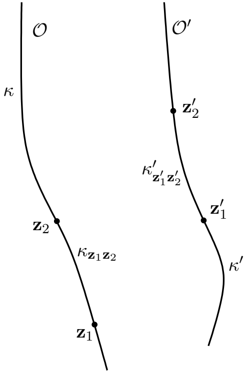


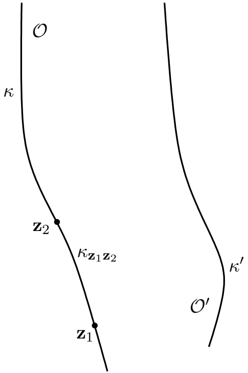
♠ We choose an interval of each world lines, measure the lapses $\Delta\tau$ and $\Delta\tau'$ along the intervals and define the time dilation ratio

$$D = \frac{\Delta\tau}{\Delta\tau'}$$

with time dilation being equivalent to $D \neq 1$.

Interval pairing—the standard method

♠ $\mathcal{O}$ at $\mathbf{z}_1$ and $\mathbf{z}_2$ sends to $\mathcal{O}'$ light flashes —the corresponding null geodesics define the interval $\kappa'_{\mathbf{z}'_1\mathbf{z}'_2}$ of the world line κ' of $\mathcal{O}'$.



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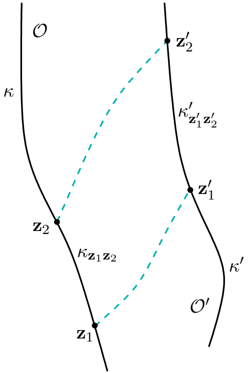

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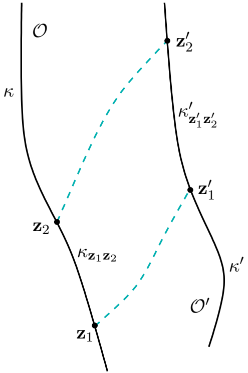

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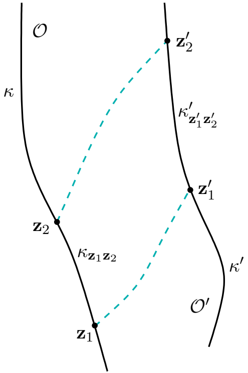

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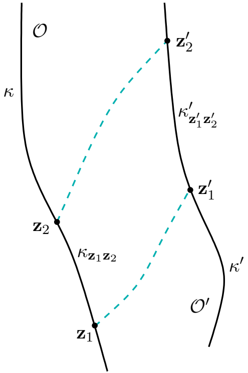

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♠ In a general spacetime gravitational time dilation is not uniquely defined.

Killing observers

♠ A spacetime metric g is *stationary*, if there exists a timelike Killing vector field K of g:

$$0 > g(K, K) \equiv g_{KK}, \qquad \mathcal{L}_K g = 0.$$

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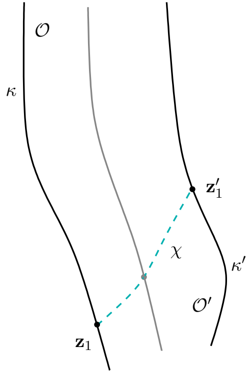
♠ Two observers $\mathcal{O}$ and $\mathcal{O}'$ will be called *Killing observers*, if their world lines are integral curves of the same Killing vector f. K.

Time dilation between Killing observers



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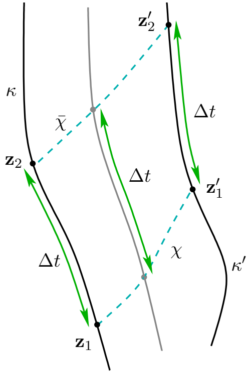
- We obtain $\bar{\chi}$ by shifting χ by Δt along the integral curves of K.



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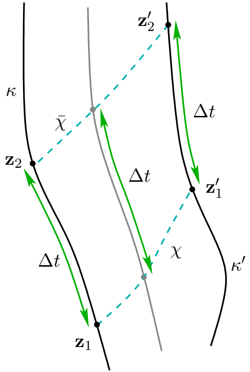

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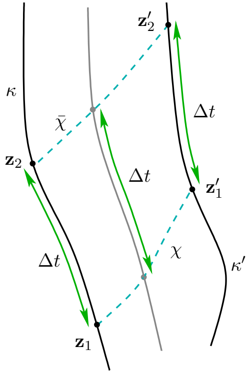
because $\mathcal{L}_K\big(g(K,K)\big) = 0$.



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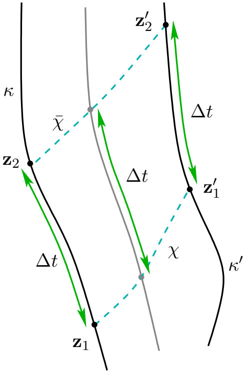

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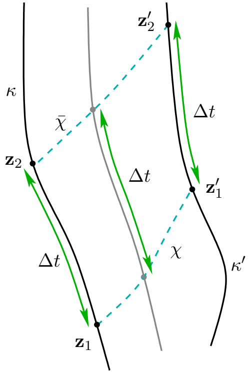

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- *Time slows down as $|g_{KK}|$ decreases.*

Gravity strength

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♠ Therefore I will use the Kretschmann scalar

$$\mathcal{R}^2 := R_{\alpha\beta\mu\nu} R_{\alpha'\beta'\mu'\nu'} g^{\alpha\alpha'} g^{\beta\beta'} g^{\mu\mu'} g^{\nu\nu'}.$$

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♠ *Does $|g_{KK}|$ always decrease as $\mathcal{R}^2$ increases?* or *Are $|g_{KK}|$ and $\mathcal{R}^2$ always negatively correlated?*

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- But if $A\mathcal{R}^2 \geq 0$, then time 'slows down' as gravity strength measured by $\mathcal{R}^2$ *is constant or decreases*.

Static observers in the Schwarzschild spacetime

- The metric

$$g = -\left(1 - \frac{2M}{r}\right)dt^2 + \left(1 - \frac{2M}{r}\right)^{-1}dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2),$$

is static, if $r > 2M > 0$ with $K = \partial_t$.

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♠ Thus here time 'slows down' as gravity strength measured by $\mathcal{R}^2$ increases, as expected.

Orbiting observers in the Schw. spacetime with $M > 0$

♠ If $\omega \neq 0$ is a constant, then $\bar{K} = \partial_t + \omega\partial_\varphi$ is a Killing vector field of the Schwarzschild metric. $\bar{K}$ is timelike on the 'equatorial plane' $\theta = \pi/2$, if

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♠ If $\omega^2 < 1/(27M^2)$, then W has three real roots r_1, r_2 and r_3 such that

$$r_1 < 2M < r_2 < r_0 < r_3,$$

where $r_0 \equiv \sqrt[3]{M/\omega^2} > 3M$ is a zero of the four-acceleration

$$\bar{A} = \frac{1}{|g_{\bar{K}\bar{K}}|}\left(1 - \frac{2M}{r}\right)\left(\frac{M}{r^2} - \omega^2 r\right)\partial_r.$$

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♠ Thus for orbiting observers of $r \in]r_0, r_3[$ and $\theta = \pi/2$ time *'speeds up'* as gravity strength measured by $\mathcal{R}^2$ increases.

The Schwarzschild–de Sitter spacetime

- $K = \partial_t$ is a Killing vector field of

$$g = -\left(1 - \frac{2M}{r} - \frac{\Lambda}{3} r^2\right) dt^2 + \left(1 - \frac{2M}{r} - \frac{\Lambda}{3} r^2\right)^{-1} dr^2 + r^2 d\Omega^2.$$

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- For $M, \Lambda > 0$ the case reduces to the previous one:

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$\mathcal{R}^2 = 48M^2/r^6$,	$\mathcal{R}^2 = \mathrm{const.} + 48M^2/r^6$.

- If $0 < \Lambda < 1/(9M^2)$, then for some *static* observers in the Schw.–de Sitter spacetime time *'speeds up'* as gravity strength measured by $\mathcal{R}^2$ increases.

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(since $A|g_{KK}| > 0$ and $\bar{A}|g_{\bar{K}\bar{K}}| > 0$).

♠ With regard to the class of all possible Killing observers, the statement *'time always slows down as gravity increases'* is false, if gravity strength is defined by means of any *observer independent* scalar field derived from the spacetime metric.

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there is no time dilation between observers given by $K = \partial_t$, because $g_{KK} = -1$ is constant. But the metric (1) is not flat.

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- Perhaps it would be better, if time dilation in GR was called *accelerational time dilation*.

References

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