FACULTY OF PHYSICS
UNIVERSITY OF WARSAW

One-dimensional Heisenberg model solution with the use of Bethe ansatz

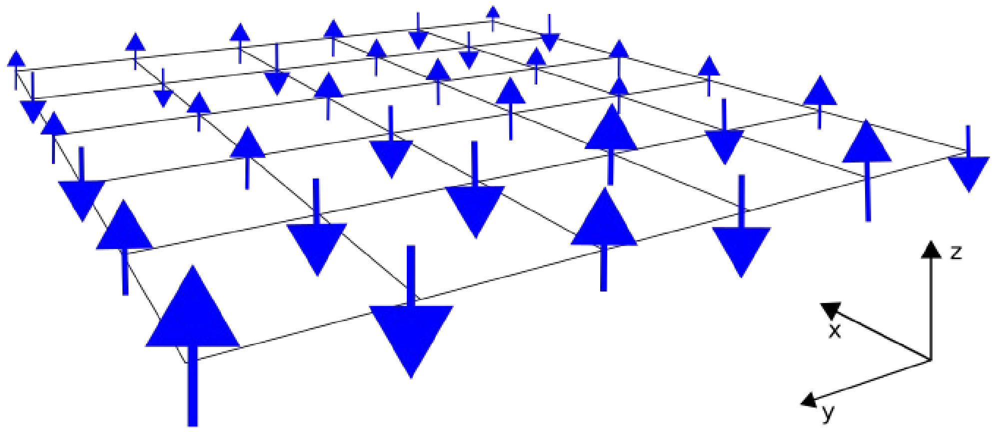
Jakub Grabarczyk
bachelor thesis under supervision of
dr hab. Jerzy Jacek Wojtkiewicz, prof. UW

Main goal of the work

Comparison results from exact diagonalization of XXZ Heisenberg chain with Bethe ansatz solution.

Heisenberg model

- Proposed in 1928 by Heisenberg as a way of describing magnetic phenomenon in solids.



Werner Heisenberg
1901 - 1976

XYZ chain Hamiltonian

$$H = -\sum_{n=1}^{N} \left(J_x S_n^x S_{n+1}^x + J_y S_n^y S_{n+1}^y + J_z S_n^z S_{n+1}^z \right)$$

J_x, J_y, J_z - energy units

XXZ case:

$$H = -\sum_{n=1}^{N} \left(S_n^+ S_{n+1}^- + S_n^- S_{n+1}^+ + 2\Delta \left(\frac{1}{4} - S_n^z S_{n+1}^z \right) \right)$$

S_n^+, S_n^- - ladder operators

Bethe ansatz (1931)

$$\psi_M = \sum_{\{n\}} a(\{n\}) \, |n_1, n_2, ..., n_M\rangle$$

$$a(n_1, n_2, ..., n_M) = \sum_{P \in S_M} \mathrm{sign}(P) A(P) \exp\left(i \sum_{j=1}^{M} k_{P_j} n_j \right)$$

$$E = 2 \sum_{j=1}^{M} (\Delta - \cos(k_j))$$

Bethe equations:

$$N k_j = 2\pi I_j + \sum_{\substack{l=1 \\ l \neq j}}^{M} \theta_{jl} \qquad e^{-i\theta_{jl}} = \frac{e^{i(k_j + k_l)} - 2\Delta e^{ik_l} + 1}{e^{i(k_j + k_l)} - 2\Delta e^{ik_j} + 1}$$

Hans Bethe 1906 - 2005

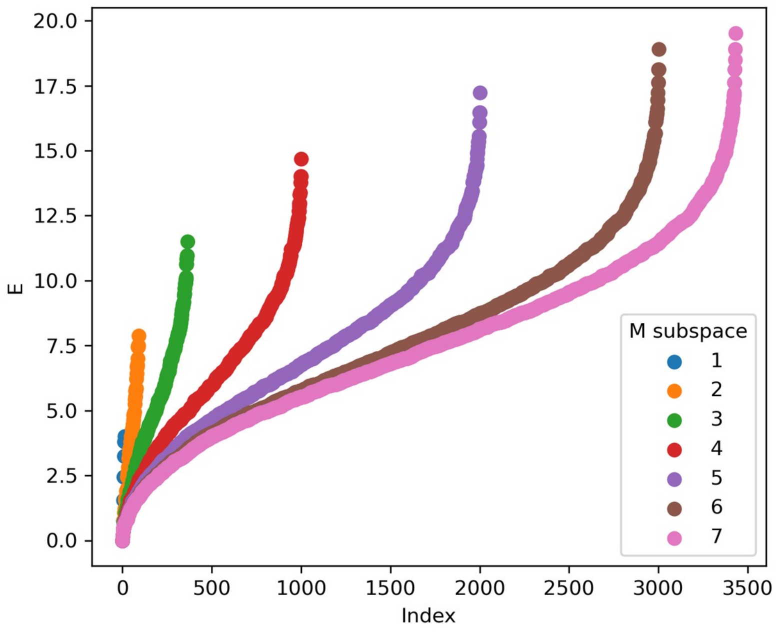
Bethe equations solution

- Allows to obtain complex quasimomenta k_j, and with them all thermodynamical properties of the system.
- Reduction of size to linear in M, but equations becomes nonlinear with $\binom{N}{M}$ solutions.
- Both algebraic and numerical approaches.

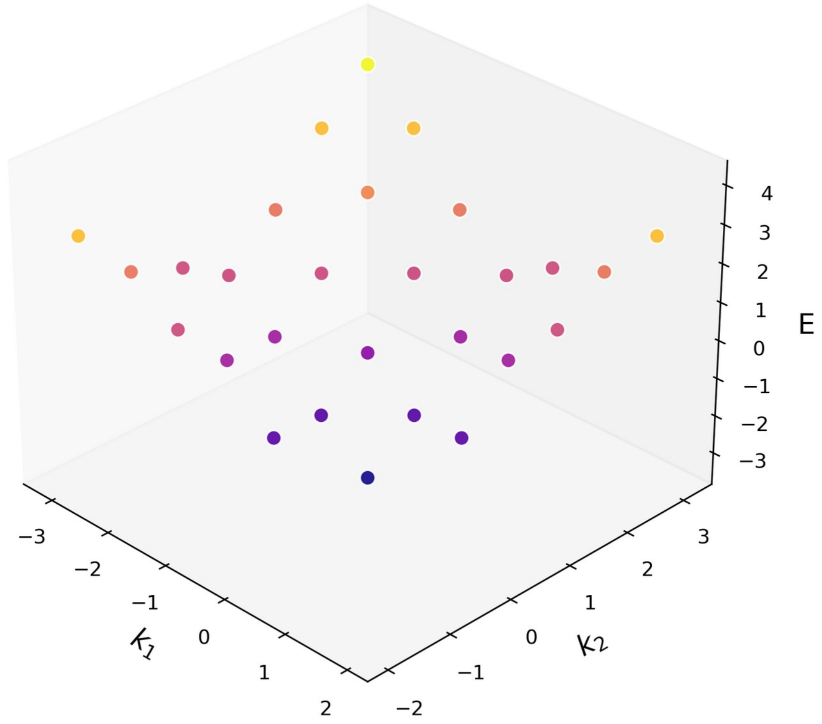
Exact diagonalization

- Gives energies of the system, so partition function and other thermodynamical quantities.
- Diagonalization of matrix for each M-subspace.
- Exponential complexity in amount of sites.

Results



Matrix eigenvalues for Δ = 1,
N = 14 spins and various M



Bethe ansatz solutions for Δ = 0.1,
N = 8 spins and M = 2

Summary

- Heisenberg model in 1D can be solved analytically with Bethe ansatz nonlinear equations.
- Exact diagonalization problem grows exponentially with number of sites.

Thank you for your attention