Stokes flow around a sphere with odd viscosity

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Collaboration

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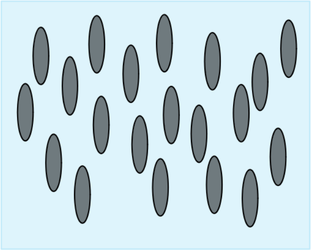
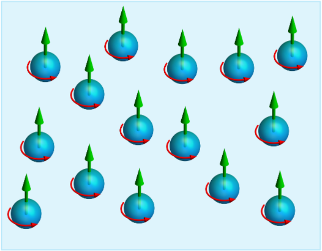
Everts, J. C., & Cichocki, B. (2024). **Dissipative Effects in Odd Viscous Stokes Flow around a Single Sphere.** Physical Review Letters, 132(21)

Introduction and motivation

- We are interested in **anisotropic** fluids:





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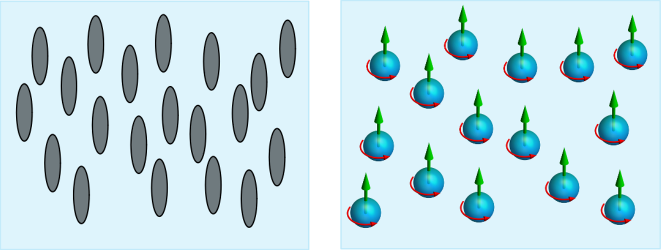

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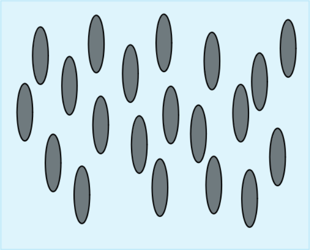

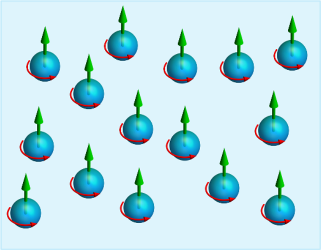

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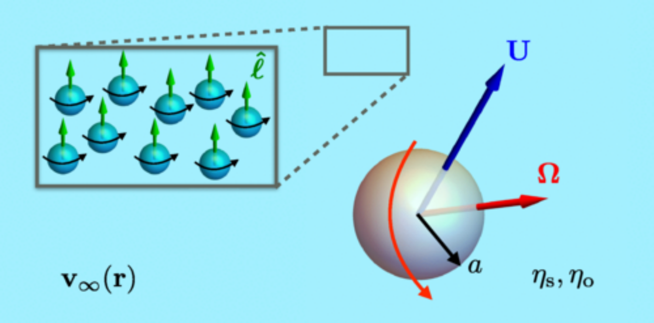
- Antisymmetric part of viscosity tensor $\implies$ **odd viscosity**

Question: How does odd viscosity affect the flow around a sphere?



Stokes flow around sphere

Linear momentum balance for incompressible, steady flow in low *Re* regime:

$$\begin{cases} \nabla \cdot \boldsymbol{\sigma} = \mathbf{0}, \\ \nabla \cdot \mathbf{v} = \mathbf{0} \end{cases} + \text{No-slip boundary conditions}$$

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Model fluid with one shear and one odd viscosity η_s, η_o

$$\eta_s \nabla^2 \mathbf{v}(\mathbf{r}) - \nabla \tilde{p}(\mathbf{r}) + \eta_o(\hat{\boldsymbol{\ell}} \cdot \nabla)[\nabla \times \mathbf{v}(\mathbf{r})] = \mathbf{0}$$

Flow around sphere

Solution in ordinary stokes flow: $\mathbf{v}(\mathbf{r}) = 6\pi\eta_s a\left(1 + \frac{a^2}{6}\nabla^2\right)\mathbf{G}\cdot\mathbf{U}$

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Solution for anisotropic fluid with odd viscosity:

$\mathbf{v}(\mathbf{r}) = \mathcal{L}_0\mathbf{G}(\mathbf{r})\cdot\ \zeta^{tt}\ \cdot\mathbf{U}$,

$$\zeta^{tt} = 24\pi\eta_s a\left\{\frac{[\gamma^2 n(\gamma)+4](\mathbf{I}-\hat{\boldsymbol{\ell}}\hat{\boldsymbol{\ell}}+2\gamma f(\gamma)\boldsymbol{\epsilon}\cdot\hat{\boldsymbol{\ell}})}{4\gamma^2 f(\gamma)^2+[4+\gamma^2 n(\gamma)]^2} + \frac{\hat{\boldsymbol{\ell}}\hat{\boldsymbol{\ell}}}{2\gamma^2 m(\gamma)+4}\right\}, \quad \gamma = \frac{\eta_o}{\eta_s}$$

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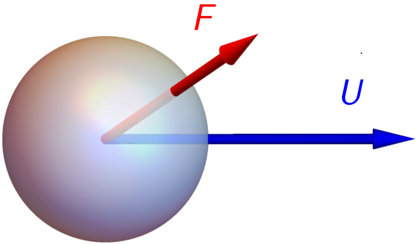
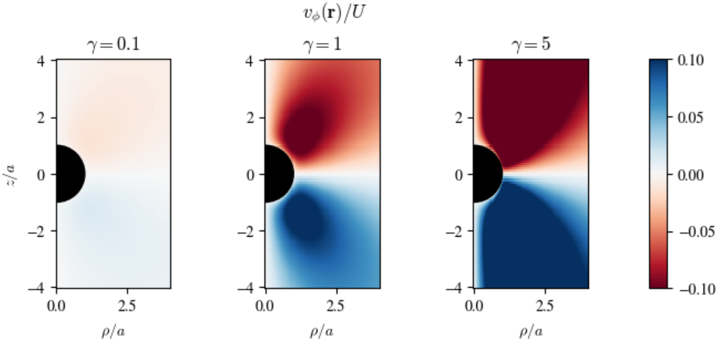
components $\perp$ **U**



Velocity field (azimuthal component)

Exact solution for $\mathbf{U} \parallel \hat{\boldsymbol{\ell}}$:

$$\mathbf{v}(\mathbf{r}) = \frac{3aU}{\gamma^2 m(\gamma) + 2}\left\{\left[\frac{\arcsin(\mathcal{R}_+^{-1})}{\gamma a \sin^2\psi} - \frac{1}{\gamma^2 r}\right]\hat{\boldsymbol{\ell}} + \frac{\hat{\boldsymbol{r}}\cdot\hat{\boldsymbol{\ell}}}{\gamma^2 r\sqrt{1-(\hat{\boldsymbol{r}}\cdot\hat{\boldsymbol{\ell}})^2}}\left(\frac{1-\mathcal{R}_-^2}{|\hat{\boldsymbol{r}}\cdot\hat{\boldsymbol{\ell}}|} - 1\right)(\gamma\hat{\boldsymbol{\phi}} - \hat{\boldsymbol{\rho}})\right\}$$



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What about $\mathbf{U} \perp \hat{\boldsymbol{\ell}}$?

Current work

1. Finding flow field around translating sphere for general direction of $\mathbf{U}$,
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Thank you for your attention!