Generalized BBGKY hierarchy for nearly-integrable systems

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Theoretical Physics Symposium, 6.12.2024

BBGKY hierarchy - a cornerstone of kinetic theory

We consider a dynamics of N classical particles $H = \sum_i \frac{p_i^2}{2m} + \frac{1}{2}\sum_{i \neq j} V(x_i - x_j)$

An exact representation of the dynamics: **Born-Bogoliubiov-Green-Kirkwood-Yvon (BBGKY) hierarchy**, formulated in 1930s-1940s

$g_n(w_1, \ldots, w_n; t)$ - n-body correlation function in phase space $w_i = (x_i, v_i)$

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$$\partial_t g_1(w_1;t) + v_1 \partial_{x_1} g_1(w_1;t) = \int dw_2 \partial_{x_1} V(x_1 - x_2)\,(\partial_{v_1} - \partial_{v_2}) g_2(w_1, w_2; t)$$ n=1

$$\partial_t g_2(w_1, w_2; t) + v_1 \partial_{x_1} g_2(w_1, w_2; t) + v_2 \partial_{x_2} g_2(w_1, w_2; t) = \partial_{x_1} V(x_1 - x_2)\,(\partial_{v_1} - \partial_{v_2})\big(g_1(w_1;t) g_1(w_2;t) + g_2(w_1, w_2; t)\big) +$$

$$\int dx_3 \Big(\partial_{x_1} V(x_1 - x_3)\,(\partial_{v_1} - \partial_{v_3}) g_1(w_1;t) g_2(w_2, w_3; t) + \partial_{x_2} V(x_2 - x_3)\,(\partial_{v_2} - \partial_{v_3}) g_1(w_2;t) g_2(w_1, w_3; t)\Big) +$$

$$\int dx_3 \big(\partial_{x_1} V(x_1 - x_3)\,(\partial_{v_1} - \partial_{v_3}) + \partial_{x_2} V(x_2 - x_3)\,(\partial_{v_2} - \partial_{v_3})\big)\big(g_1(w_3;t) g_2(w_1, w_2; t) + g_3(w_1, w_2, w_3; t)\big)\Big)$$ n=2

$$\partial_t g_3(w_1, w_2, w_3; t) = (\ldots) g_4(w_1, w_2, w_3, w_4; t)$$ n=3

...

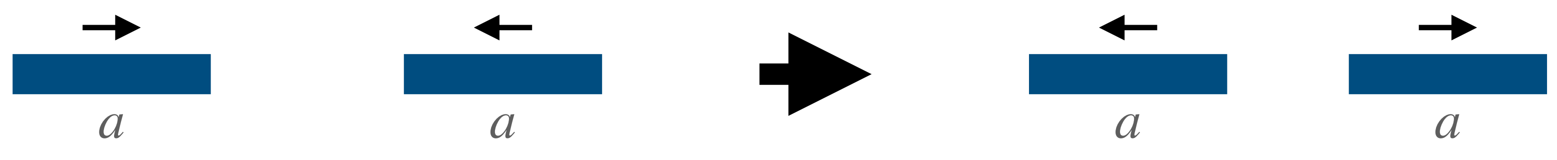
Useful for **weakly interacting systems** as $g_n \sim V^{n-1}$ and perturbative treatment is allowed.

Integrable and nearly-integrable systems

Many-particle integrable systems:

- One-dimensional
- Interacting
- Solvable, known thermodynamics (Thermodynamic Bethe Ansatz)
- Infinite number of conservation laws

Examples:



Classical: hard rod gas

$$H = -\frac{\hbar^2}{2m}\sum_i \partial_{x_i}^2 + 2c\sum_{i<j}\delta(x_i - x_j)$$

Quantum: Lieb-Liniger model

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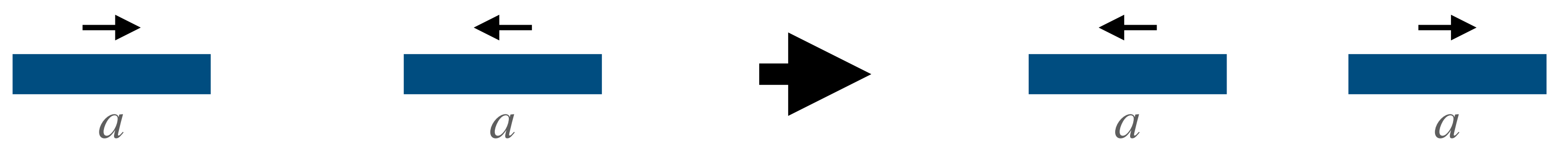

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This talk: nearly integrable models with additional, weak long-range interactions

$$H = H_{integrable} + \frac{1}{2}\int dx\, dx'\, V(x-x')\rho(x)\rho(x')$$

Main result: generalised BBGKY hierarchy

Assumptions:

- **Separation of lengthscales:** characteristic range of the potential ξ >> correlation length of unperturbed integrable model (think of hard rod diameter a)
- **Weak long range potential** $V \ll 1$, in order to truncate the hierarchy

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Correlation function structure (example of two-body function):

$$g_2(x_1, x_2) = g_2^{integrable}\delta(x_1 - x_2) + g_2^{L-R}(x_1, x_2)$$

Known

Weak O(V)

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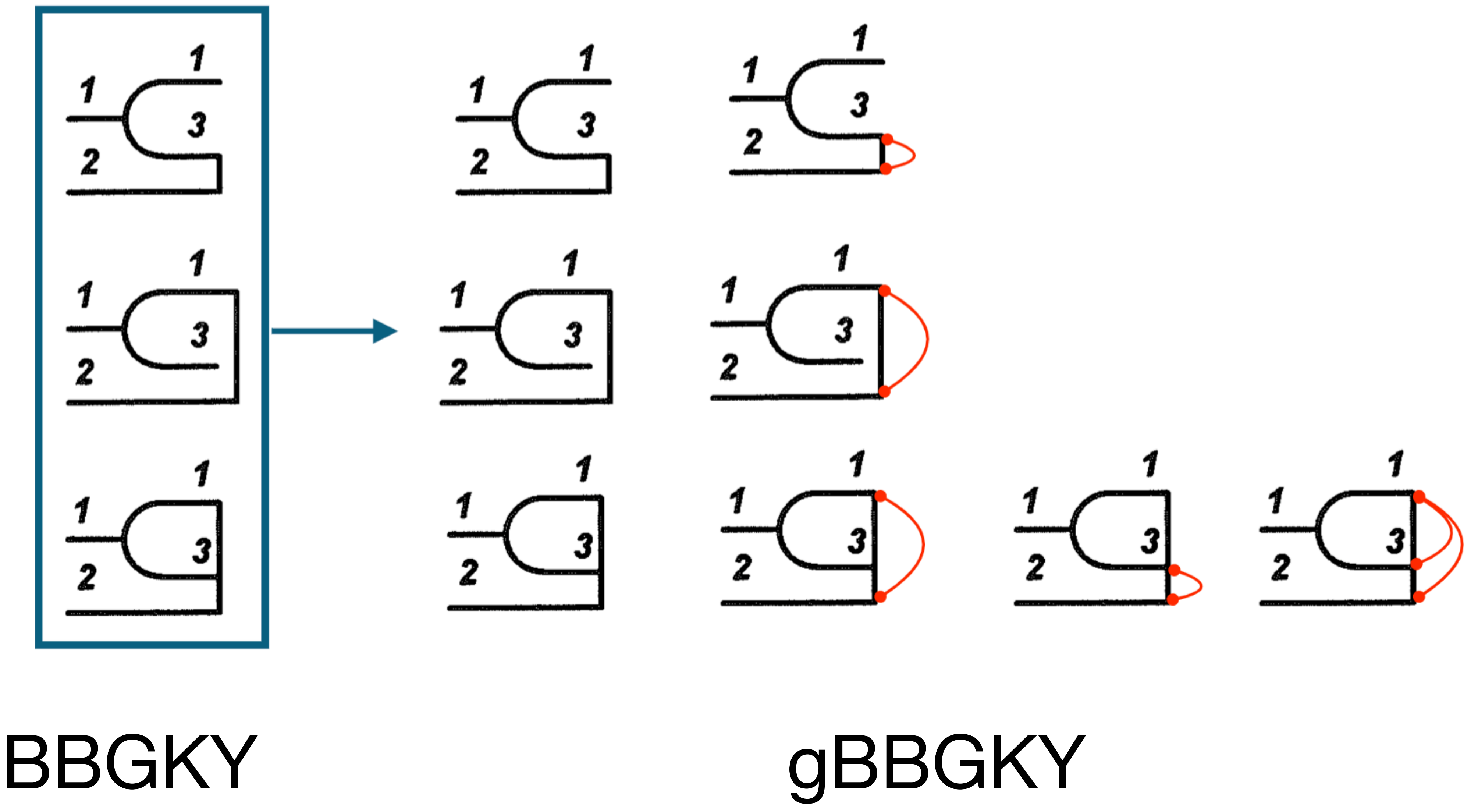
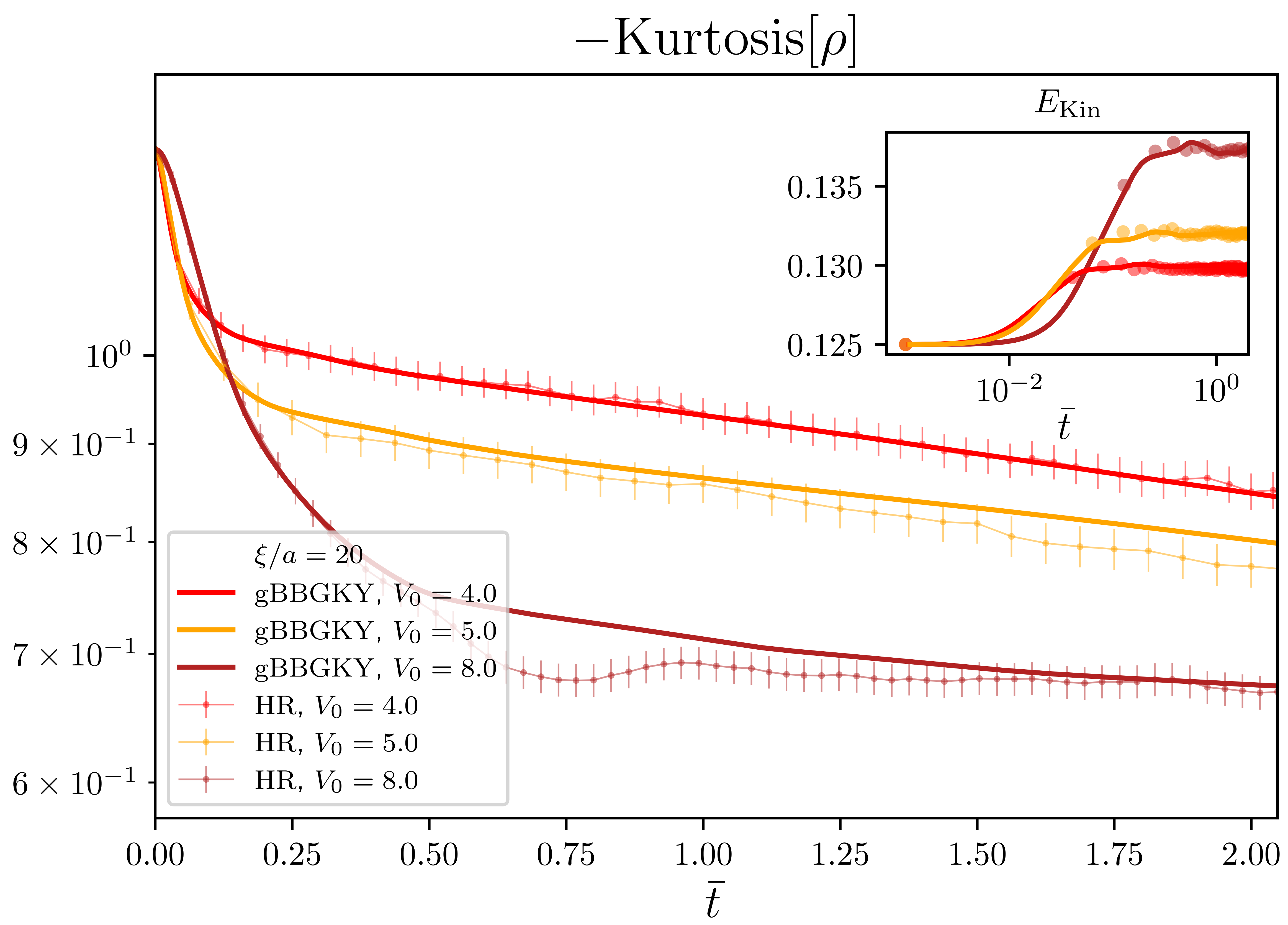
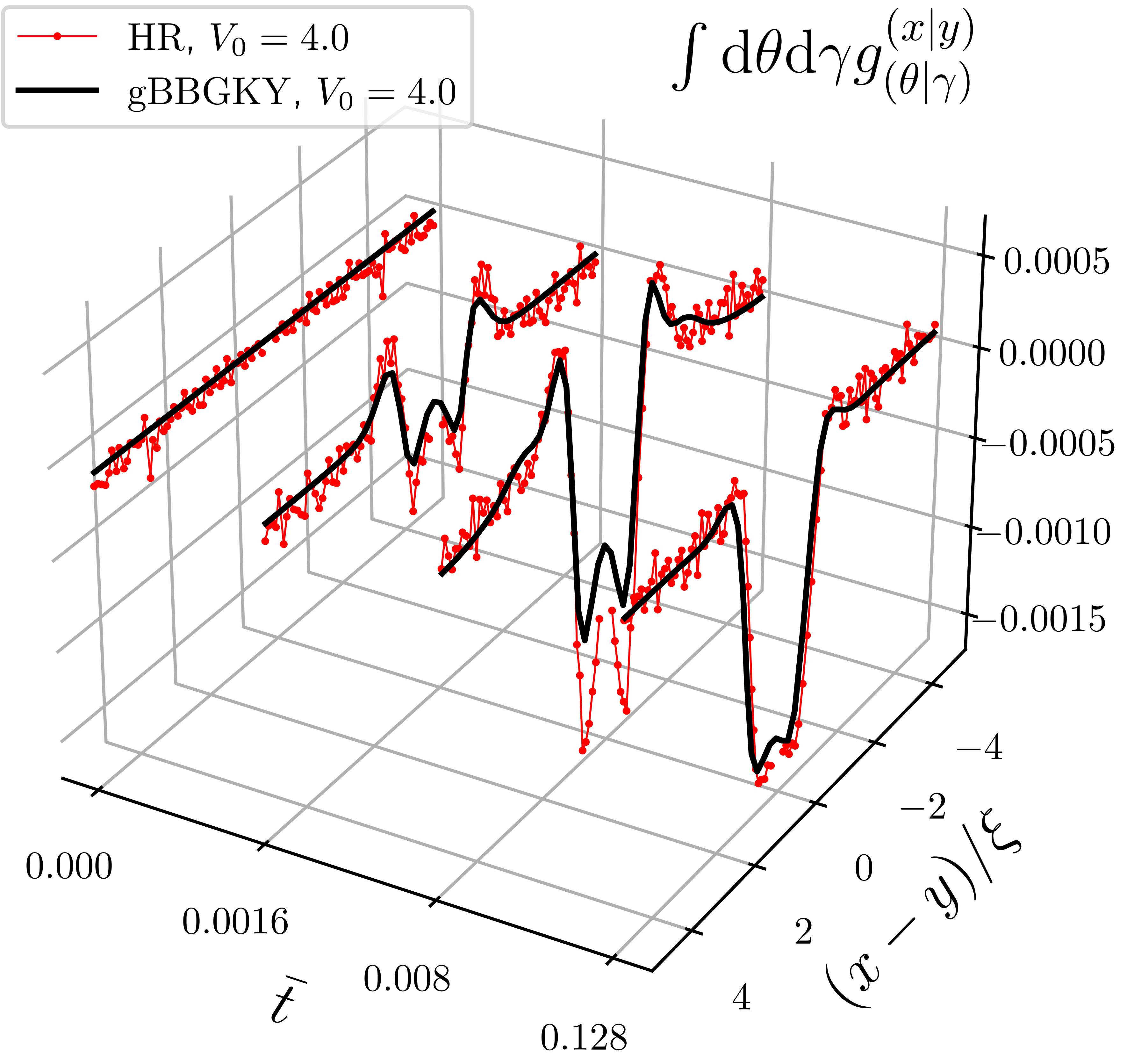
New hierarchy: similar structure but more terms (diagrams for two-body function)



Numerical benchmarks in long-range interacting hard rod gas





Thank you!