Multiloop Corrections to the Leptonic Invariant Mass Spectrum in $b \to X_c l \bar{\nu}_l$ Decay

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based on MC, M. Misiak, A. Rehman, arXiv: 2411.12866

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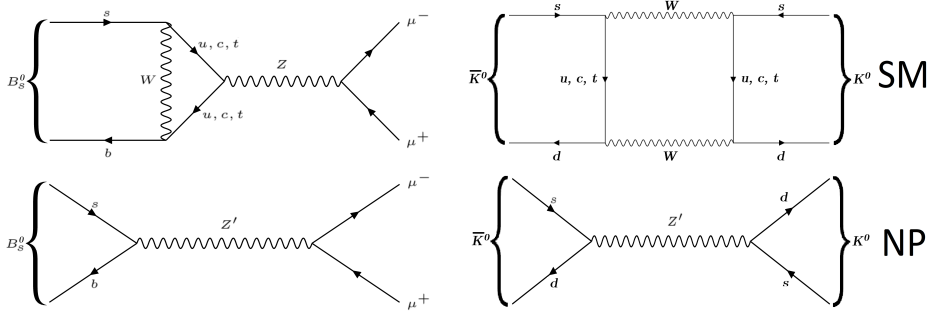
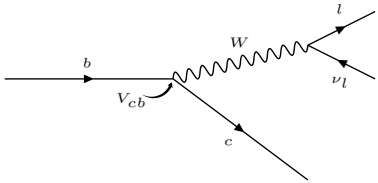
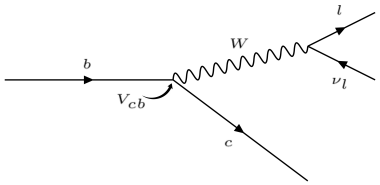
- For BSM searches, one of the most important is the $|V_{cb}|$ CKM matrix element.



- $\delta\mathcal{B}(B_s^0 \to \mu^+\mu^-) = \sqrt{\underset{|V_{cb}|}{(2.3\%)^2} + \underset{other}{(2.2\%)^2}}$
  [arXiv: 2407.03810]
- Around 50% of the theoretical error of $|\epsilon_K|$ is due to $|V_{cb}|$
  [arXiv: 2401.08006]



- The standard method of extracting the value of $|V_{cb}|$ is to study the semileptonic transition $b \to c l \bar{\nu}_l$, $l \in \{e, \mu\}$.



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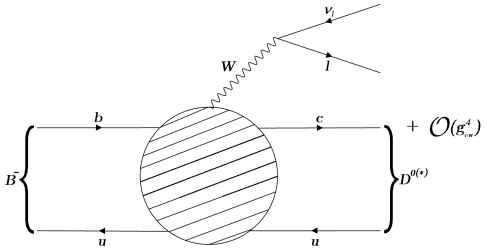
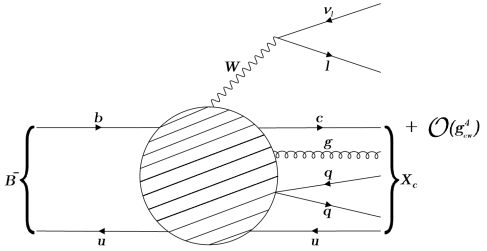
- On the hadronic level it can be realized in an exclusive or inclusive way:



- Exclusive $B^- \to D^{0(*)} l \bar{\nu}_l$ decay.
- $\mathcal{B}^{exp}(B^- \to D^* l \bar{\nu}_l) = (5.53 \pm 0.22)\%$



- Inclusive $B^- \to X_c l \bar{\nu}_l$ decay. All final states with $C = 1$ are summed over.
- $\mathcal{B}^{exp}(B^- \to X_c l \bar{\nu}_l) = (10.8 \pm 0.4)\%$

- The SM prediction of $|V_{cb}|$ can be extracted from a fit of $\hat{q}^2 \equiv (p_l + p_{\tilde{\nu}_l})^2/m_b^2$ moments of the semileptonic decay to experimental data.

$$\mathcal{M}_n \equiv \int_{\hat{q}^2_{cut}}^{\left(1-\frac{m_c}{m_b}\right)^2} d\hat{q}^2 \frac{d\Gamma}{d\hat{q}^2} \hat{q}^{2n}.$$

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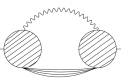
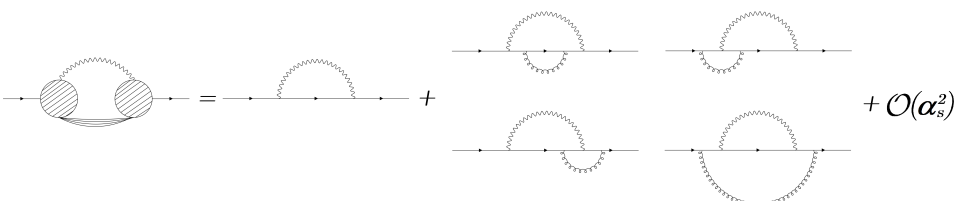
- These moments can be calculated in the Heavy Quark Expansion. The leading term corresponds to replacing the B meson with the b quark and treating QCD as perturbative.

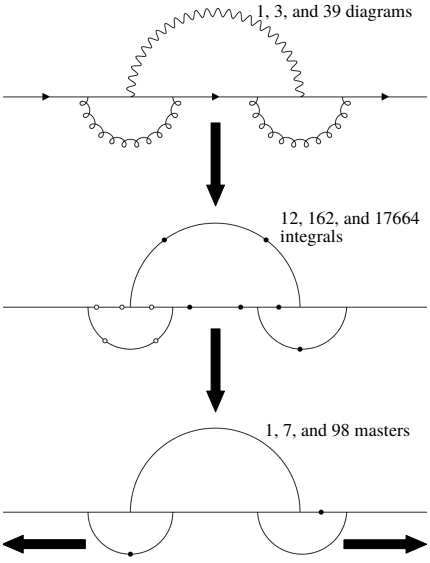
$$\frac{d\Gamma}{d\hat{q}^2} \propto \mathrm{Im}$$  $$+ \mathcal{O}\left[\frac{(m_B - m_b)^2}{m_b^2}\right]$$



$$+ \mathcal{O}(\alpha_s^2)$$

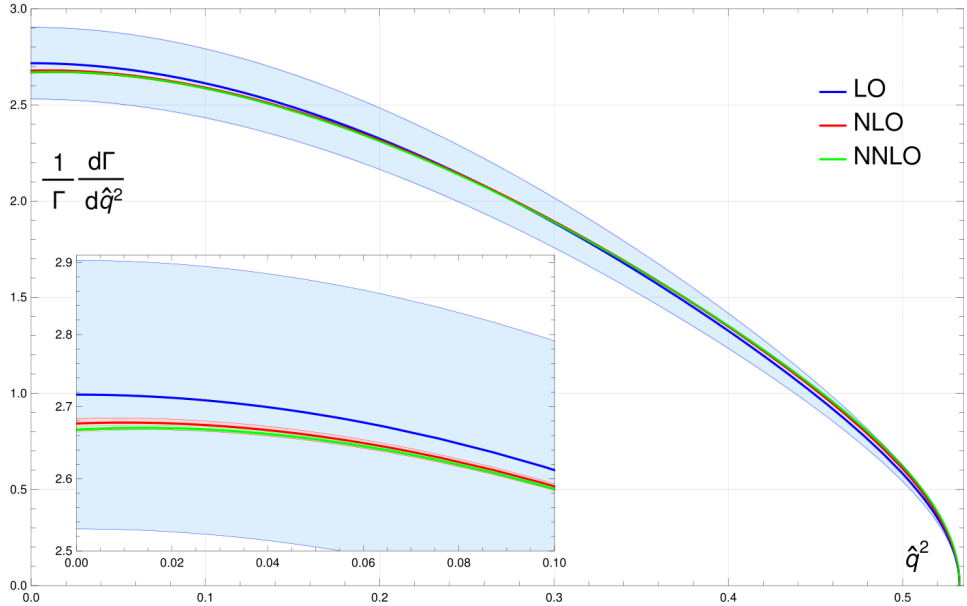
Analytic solutions:
([M. Fael and F. Herren, JHEP 05 (2024) 287])

- The DEs for a large class of integrals can be solved using the differential equations in the canonical form method.
- The boundary condition was found using `AMFlow`. [arXiv:2201.11669]
- Solution given in terms of Goncharov Polylogarithms.
- No analytic solution known for integrals with 3 cut charm quarks.



Fits to numerical solutions:
([MC, M. Misiak, A. Rehman, arXiv: 2411.12866])

- Dense scans in the (m_c, q^2) space using `AMFlow`.
- The result can be expressed using elementary functions.
- Accuracy of more than 4 significant digits when compared with exact results, far higher than experimental precision.
- Cuts through 3 charm quarks can be computed.


LO
NLO
NNLO
$\frac{1}{\Gamma}\frac{d\Gamma}{d\hat{q}^2}$
$\hat{q}^2$
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0.1
0.2
0.3
0.4
0.5
2.9
2.8
2.7
2.6
2.5
0.00
0.02
0.04
0.06
0.08
0.10

Summary

- The $|V_{cb}|$ matrix element governs the strength of interactions between bottom and charm quarks.
- Its value can be extracted from the $\hat{q}^2$ moments of the semileptonic B decay.
- The structure of the B meson is very complicated. In the leading approximation, B can be replaced by the b quark.
- The QCD corrections affect the $\hat{q}^2$ spectrum of the semileptonic decay. These are now known up to NNLO.
- We observe good convergence of the α_s perturbative series and reduction of higher order uncertainty.