Fundamental limits and optimal protocols in noisy quantum metrology

Stanisław Kurdziałek

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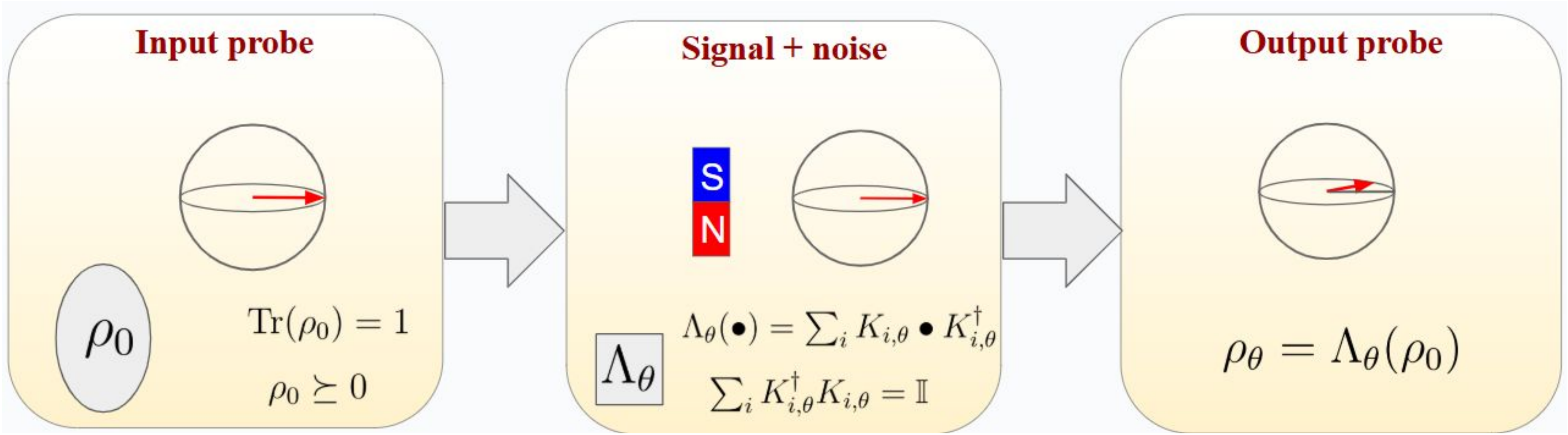
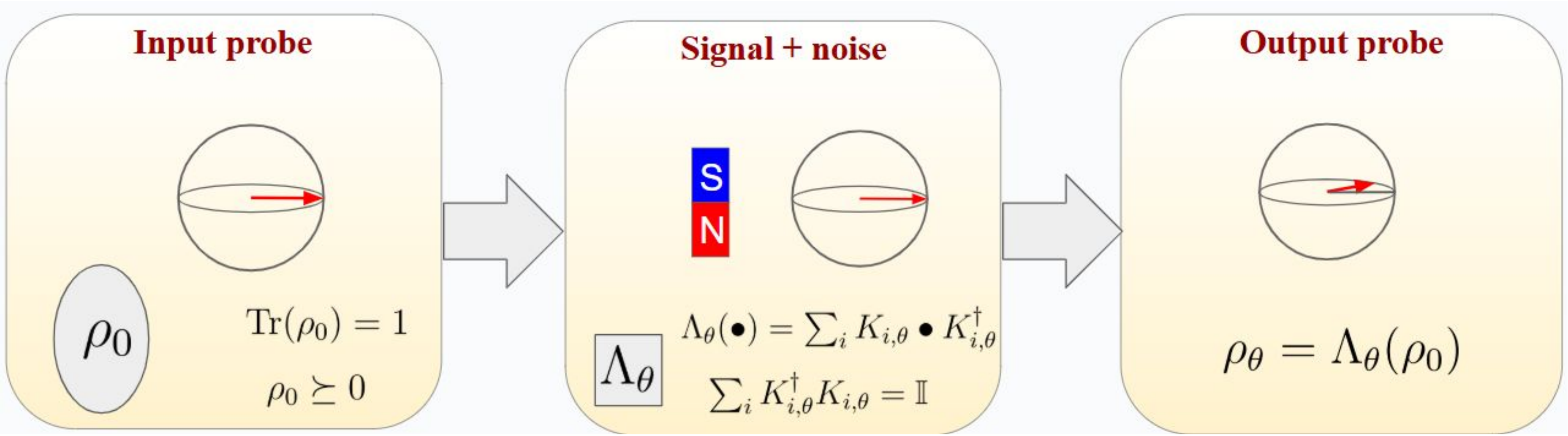
Quantum metrology: quantum channel estimation

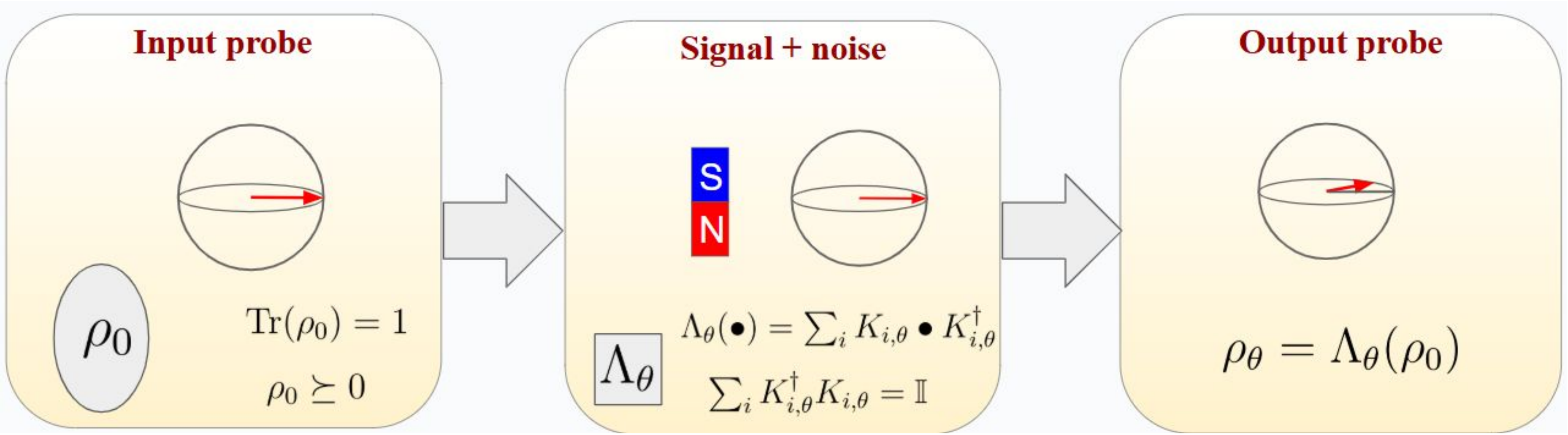


Quantum metrology: quantum channel estimation



What is the best precision of estimation?

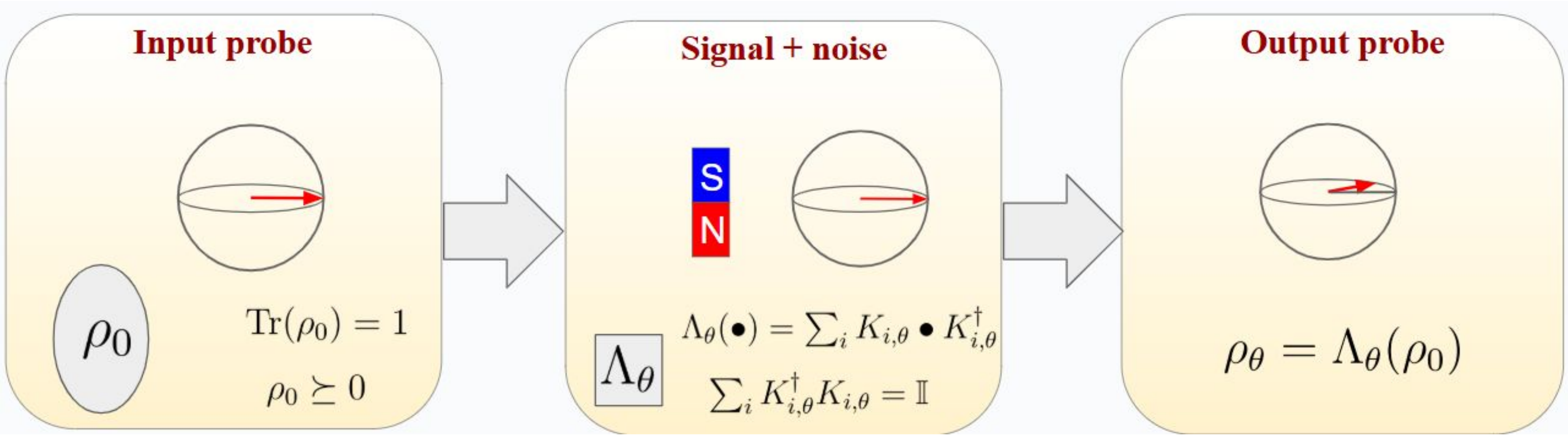
Quantum metrology: quantum channel estimation



What is the best precision of estimation?

$$\min_{\rho_0, \mathbf{M}, \tilde{\theta}} \Delta\tilde{\theta} = ?$$

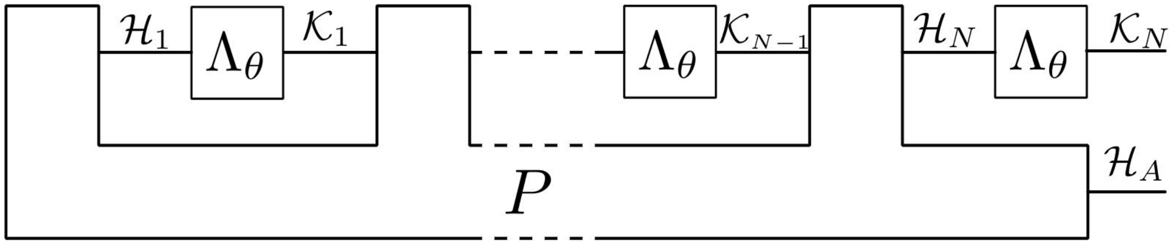
Quantum metrology: quantum channel estimation



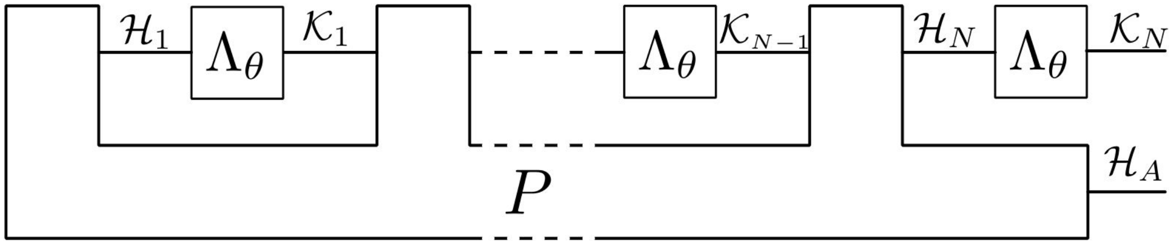
What is the best precision of estimation?

$$\min_{\rho_0, \mathbf{M}, \tilde{\theta}} \Delta\tilde{\theta} =? \qquad \max_{\rho_0} F(\Lambda_\theta(\rho_0)) =?$$

Quantum metrology: quantum channels estimation



Quantum metrology: quantum channels estimation



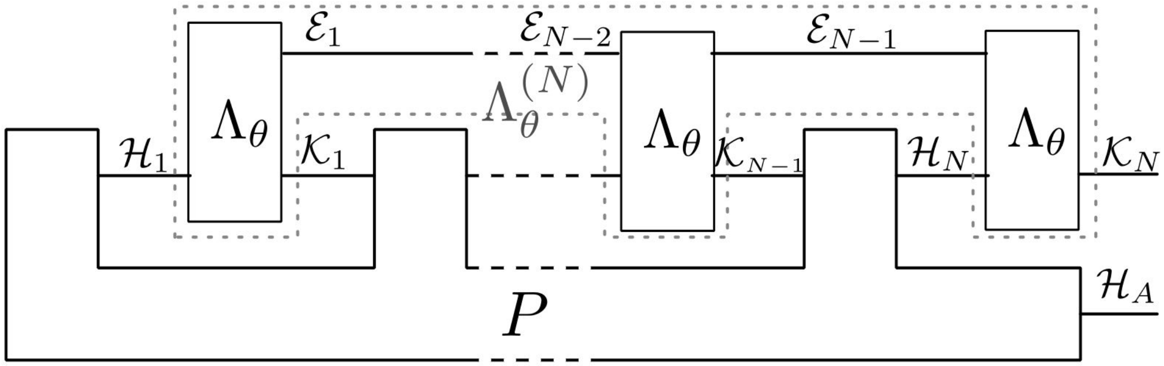
Choi-Jamiołkowski operator of a strategy comb satisfies:

$$P \in \mathrm{Lin}\left(\mathcal{H}_1 \otimes \ldots \otimes \mathcal{H}_N \otimes \mathcal{H}_A \otimes \mathcal{K}_1 \otimes \ldots \otimes \mathcal{K}_{N-1}\right)$$

$$P \geq 0, \; \mathrm{Tr}_{\mathcal{H}_A \otimes \mathcal{H}_N} P = P^{(N-1)} \otimes \mathbb{1}_{\mathcal{K}_{N-1}},$$

$$\underset{1<k<N}{\forall} \mathrm{Tr}_{\mathcal{H}_k} P^{(k)} = P^{(k-1)} \otimes \mathbb{1}_{\mathcal{K}_{k-1}}, \; \mathrm{Tr}_{\mathcal{H}_1} P^{(1)} = 1$$

Quantum metrology: quantum channels estimation



For correlated noise: $\Lambda_\theta^{(N)} \neq \Lambda_\theta^{\otimes N}$

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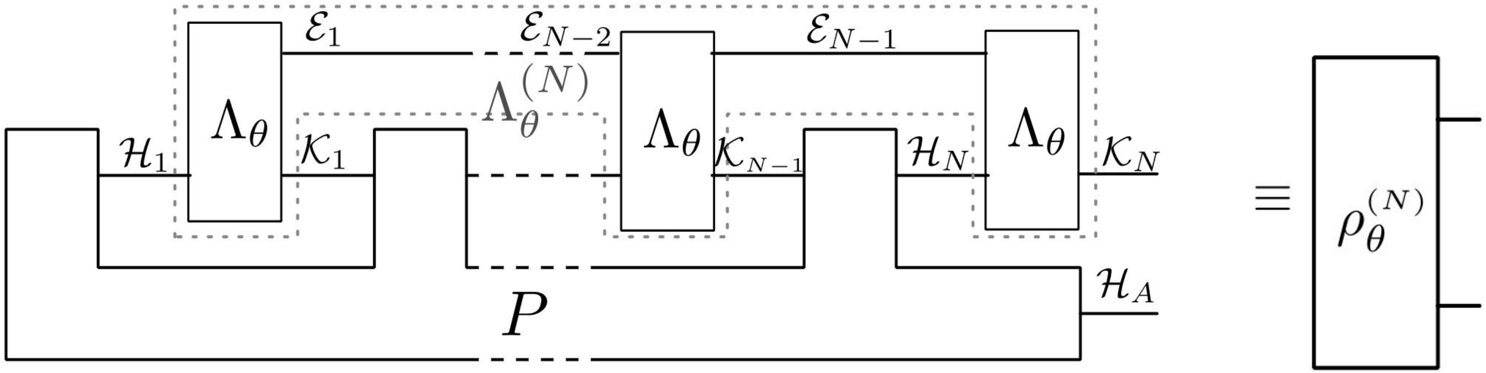
Optimization of Fisher information over combs



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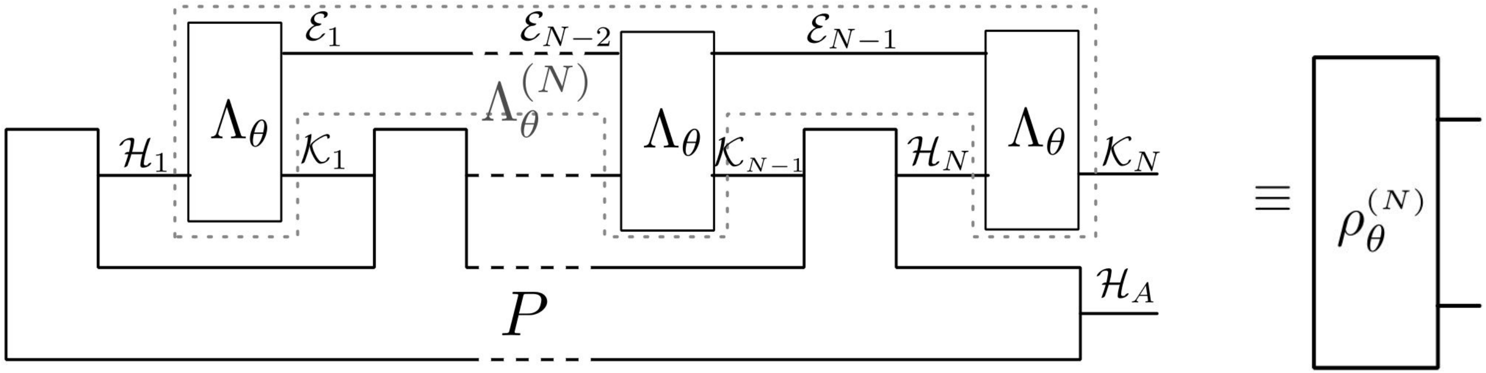

$$\rho_\theta^{(N)} = \Lambda_\theta^{(N)} * P$$

Link product, linear in both arguments (index contraction)

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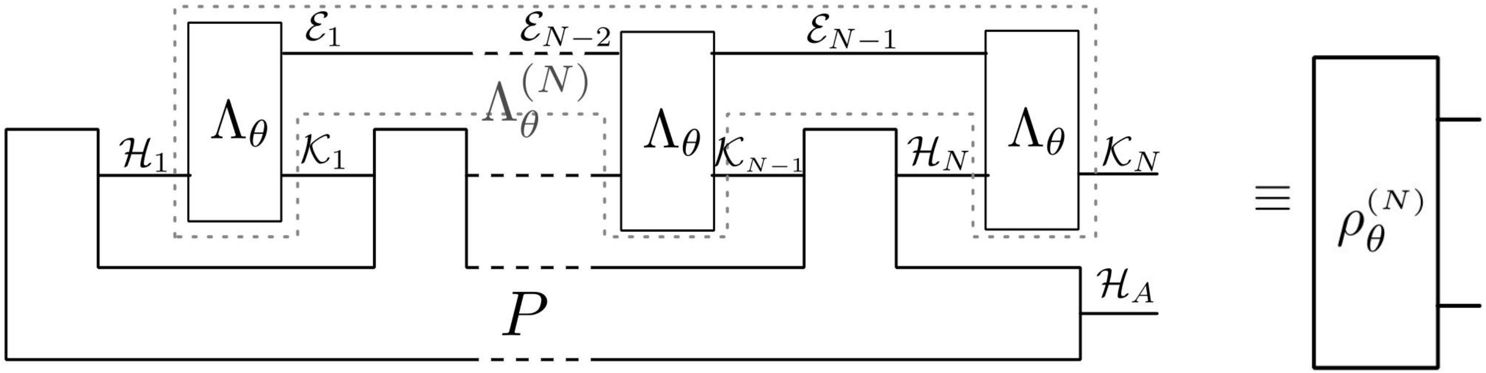

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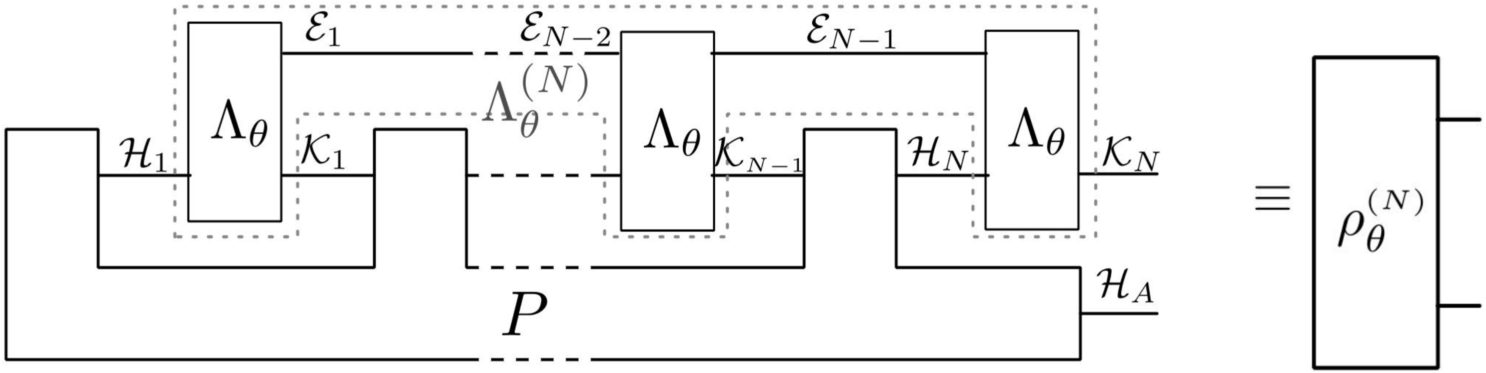

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A. Altherr, Y. Yang, PRL 127, 060501 (2021)

Q. Liu, Z. Hu, H.Yuan, Y. Yang, PRL 130, 070803 (2023)

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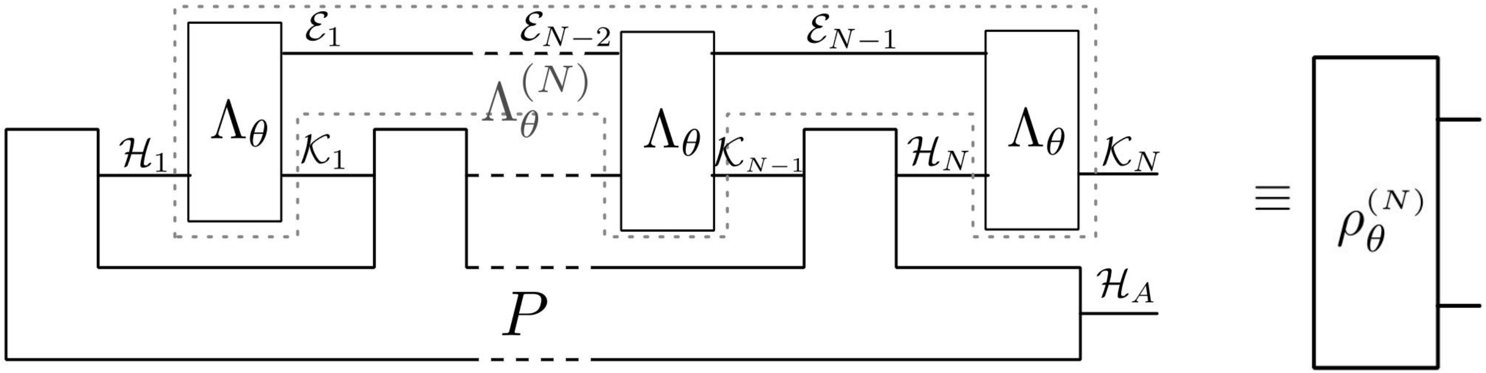

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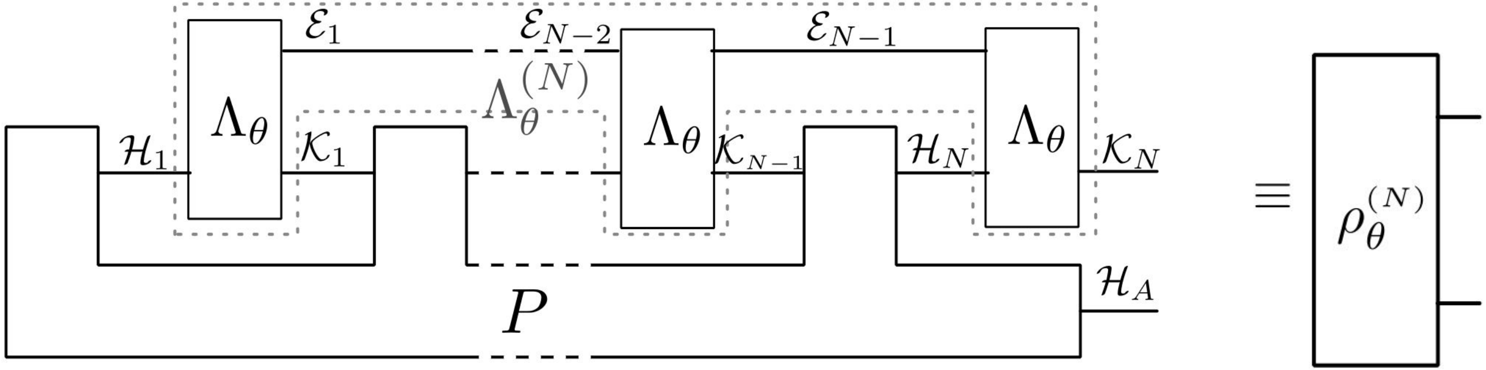

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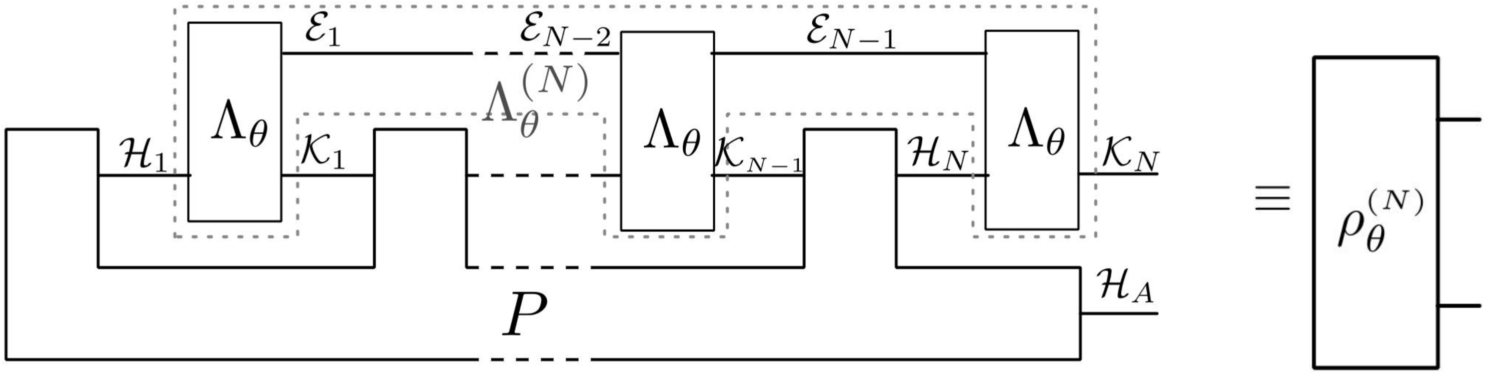
N >> 1

Go and no-go theorems

Optimization of Fisher information over combs



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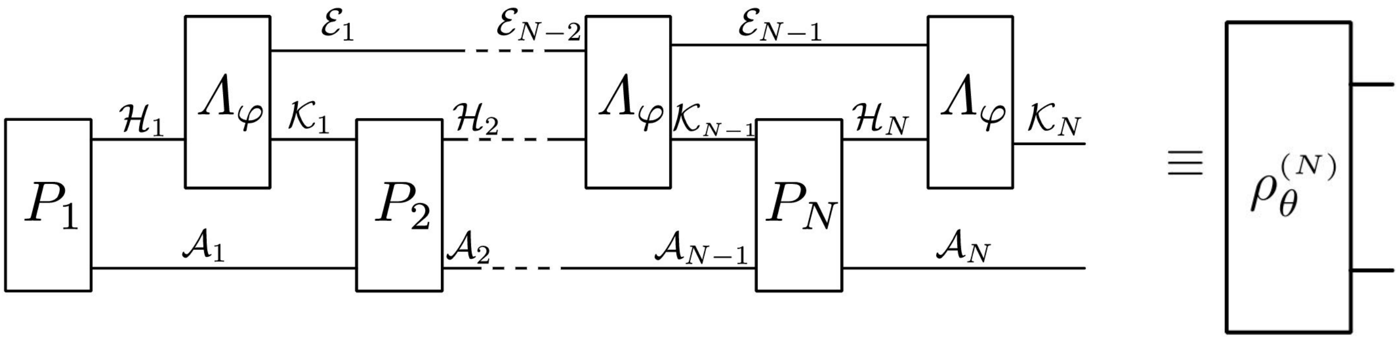
Go and no-go theorems

Our idea: Break comb into teeth



$$P = P_1 * P_2 * \ldots * P_N$$

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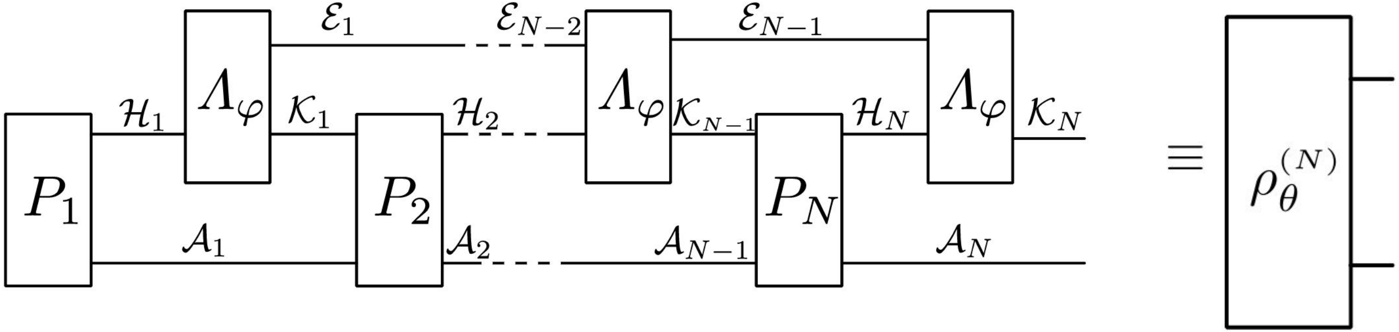

$$P = P_1 * P_2 * \ldots * P_N$$

$$d_{\mathcal{A}_i} = d_{\mathcal{A}}$$

$$d_{\mathcal{H}_i} = d_{\mathcal{K}_i} = d_{\mathcal{H}}$$

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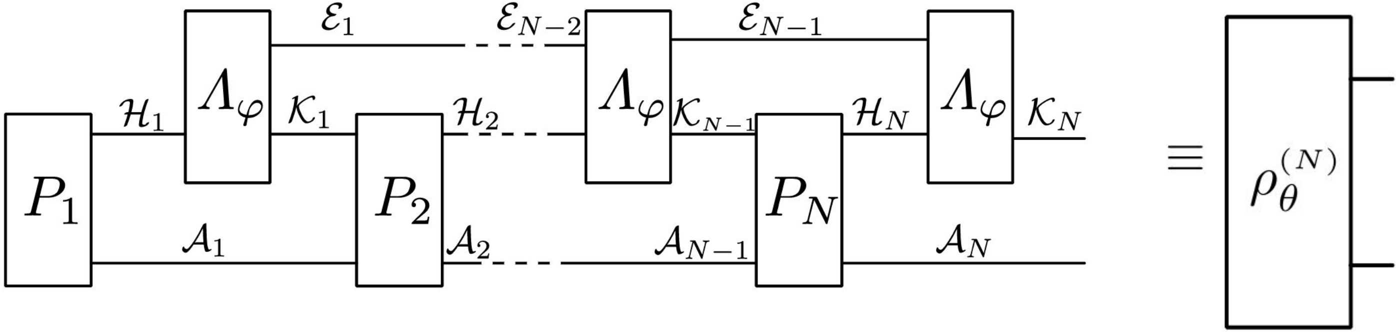

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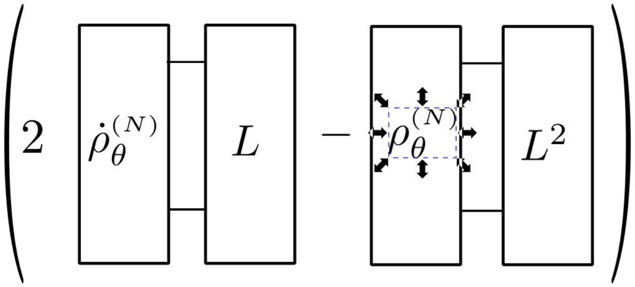
Storing P requires $d_{\mathcal{H}}^{4N-2} d_{\mathcal{A}}^2$ variables

Storing $P_1 * P_2 * \ldots * P_N$ requires $d_{\mathcal{H}}^2 d_{\mathcal{A}}^2 + (N-1) d_{\mathcal{H}}^4 d_{\mathcal{A}}^4$ variables

Exponential gain for limited ancilla size!

ITERATIVE OPTIMIZATION WITH TENSOR NETWORKS

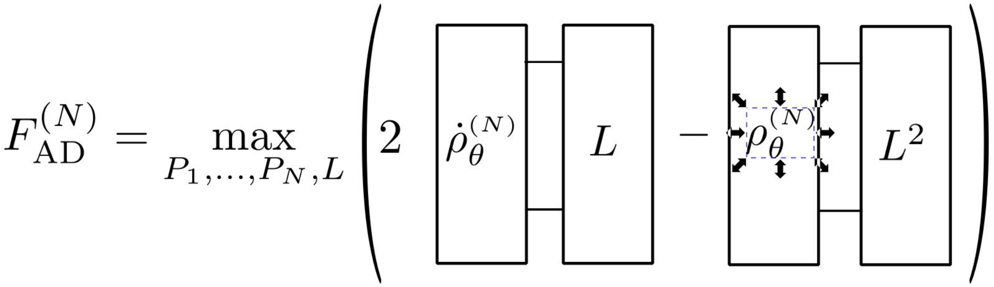
$$F_{\mathrm{AD}}^{(N)} = \max_{P_1,\ldots,P_N,L}$$



Optimization over L

Optimization over P_i

ITERATIVE OPTIMIZATION WITH TENSOR NETWORKS



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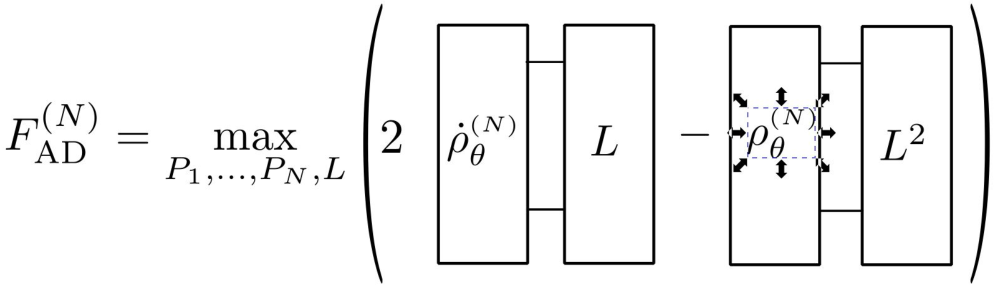
Optimization over P_i

Contract all the indices in $\rho_\theta^{(N)}, \dot{\rho}_\theta^{(N)}$ (order matters!) then solve standard SDP for L

ITERATIVE OPTIMIZATION WITH TENSOR NETWORKS

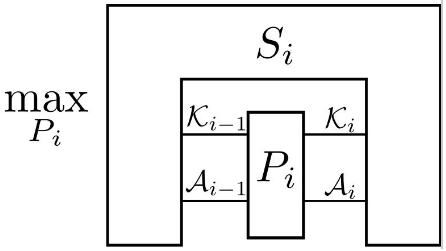


Optimization over L

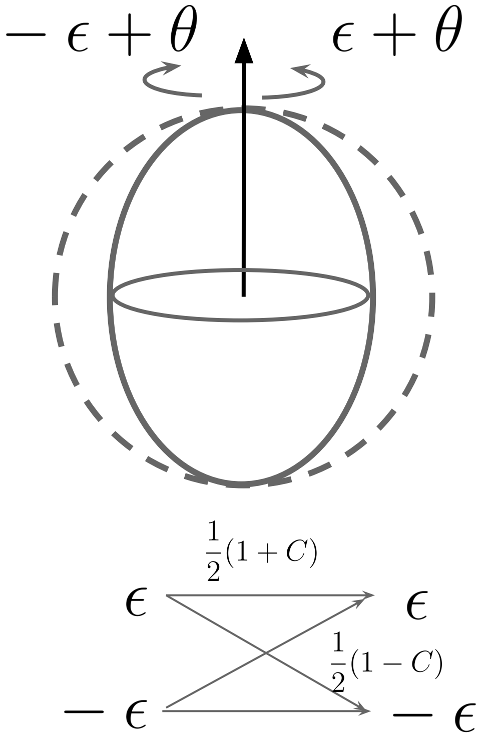
Contract all the indices in $\rho_\theta^{(N)}, \dot\rho_\theta^{(N)}$ (order matters!) then solve standard SDP for L

Optimization over P_i

Contract all the indices in the figure of merit apart from P_i indices. Then solve



Example: toy model of fluctuating magnetic field



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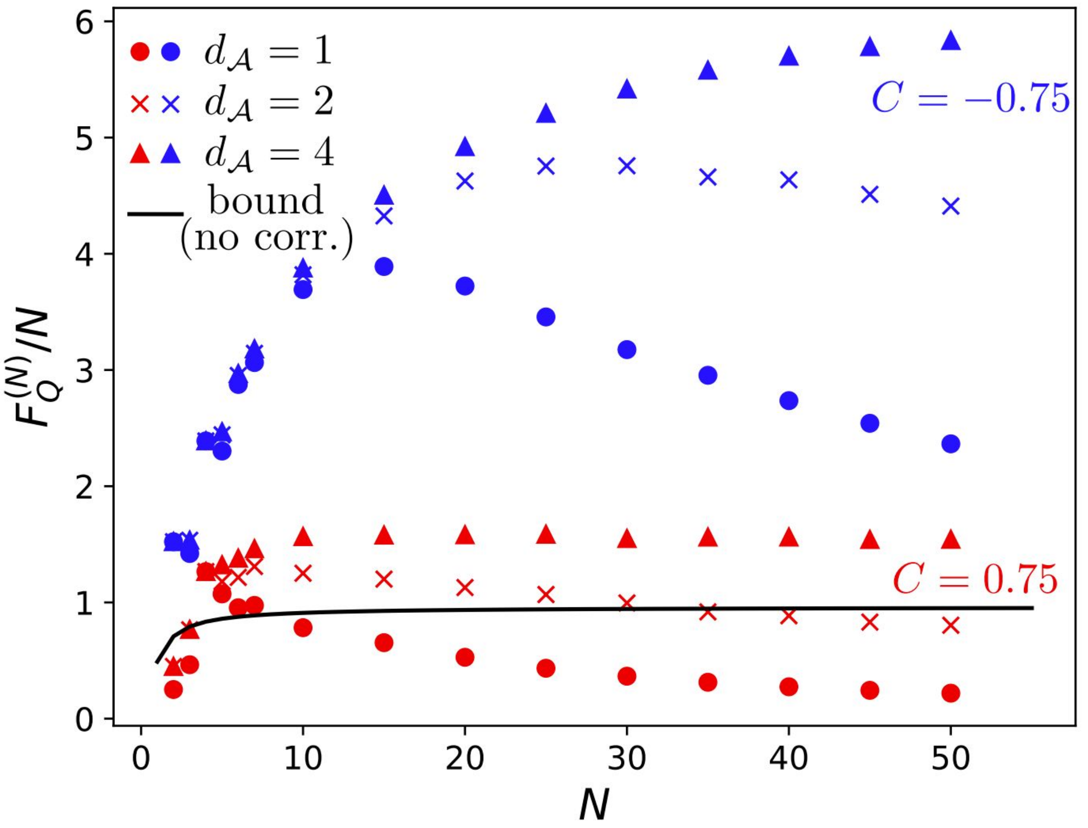

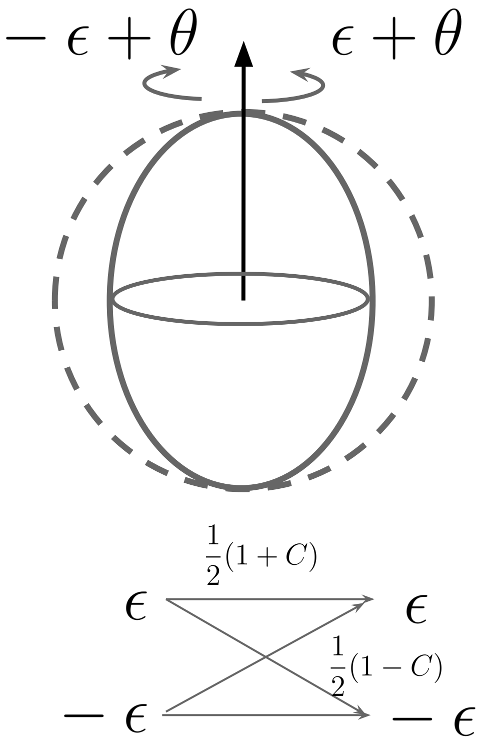

Thank you for your attention

Go theorems

arXiv:2403.04854

Quantum metrology using quantum combs and tensor network formalism

Stanisław Kurdziałek,[1] Piotr Dulian,[1,2,*] Joanna Majsak,[1,3,*]
Sagnik Chakraborty,[1,4] and Rafał Demkowicz-Dobrzański[1]

[1] *Faculty of Physics, University of Warsaw, Pasteura 5, 02-093 Warszawa, Poland*
[2] *Center for Theoretical Physics, Polish Academy of Sciences, Al. Lotników 32/46, 02-668 Warszawa, Poland*
[3] *Quantum Research Center, Technology Innovation Institute, Abu Dhabi, UAE*
[4] *Departamento de Física Teórica, Facultad de Ciencias Físicas, Universidad Complutense, 28040 Madrid, Spain*

No-go theorems

arXiv:2410.01881

Universal bounds in quantum metrology in presence of correlated noise

Stanisław Kurdziałek,[1] Francesco Albarelli,[2] and Rafał Demkowicz-Dobrzański[1]

[1] *Faculty of Physics, University of Warsaw, Pasteura 5, 02-093 Warszawa, Poland*
[2] *Scuola Normale Superiore, I-56126 Pisa, Italy*