UNIVERSITY OF WARSAW

FACULTY OF PHYSICS

Institute of Theoretical Physics

Université Paris Cité

MSC

Growth and form of transport networks

Theoretical Physics Symposium, December 5-6, 2024

Stanisław Żukowski

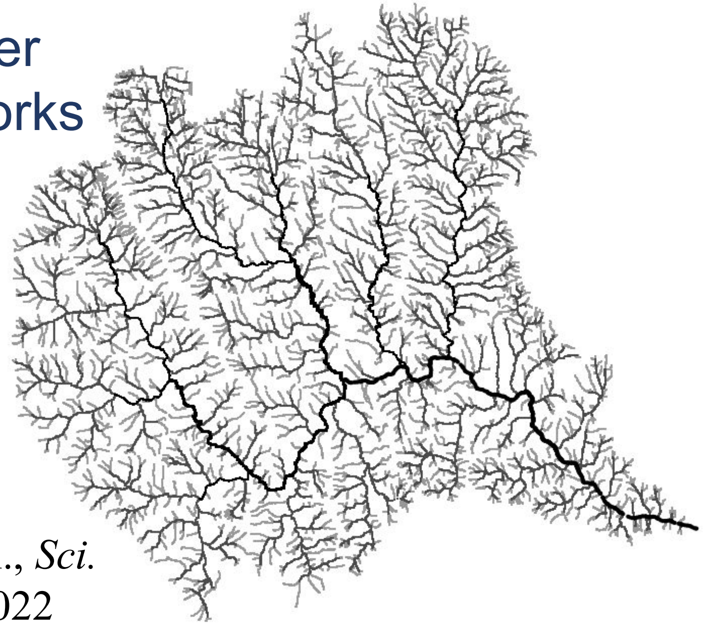
Supervisors: Piotr Szymczak, Annemiek Cornelissen

Doctoral School of Exact and Natural Sciences

EDPIF

Spatial (transport) networks

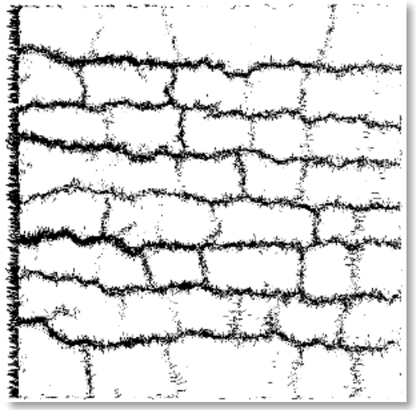
River networks



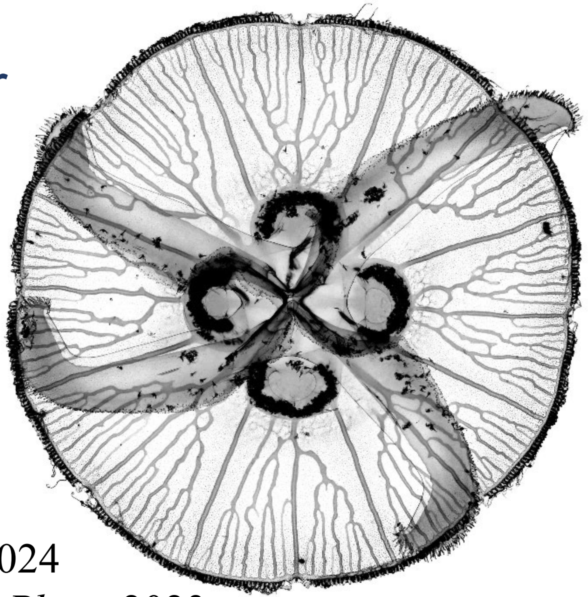
SŻ et al., *Sci. Rep.*, 2022

Propagating cracks

Maciej Kot,
Stéphane Douady



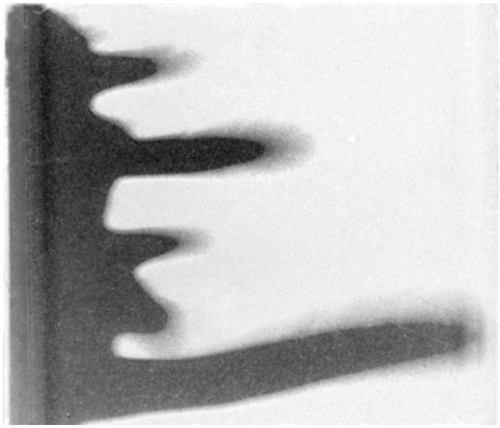
Jellyfish gastrovascular system



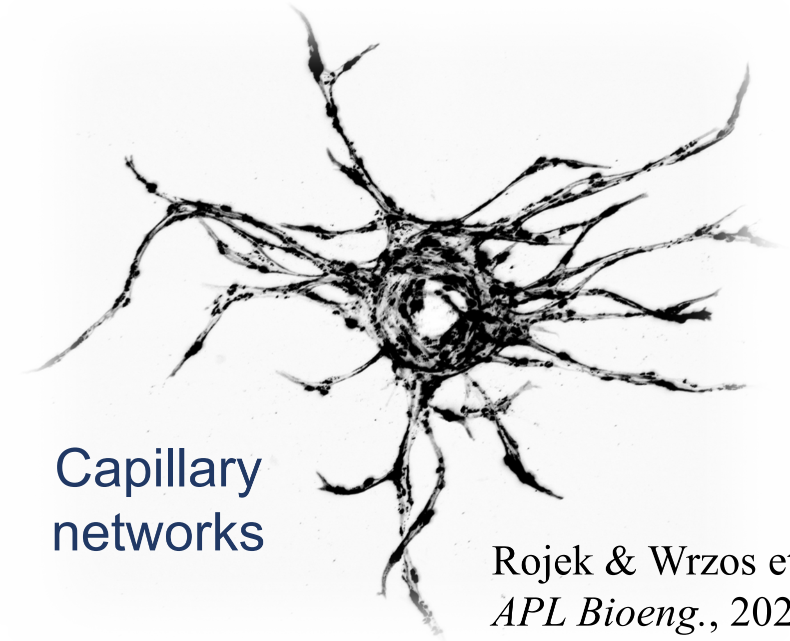
SŻ et al., *PNAS*, 2024
Song et al., *Front. Phys.*, 2023

Microfluidic experiments: fracture dissolution

Florian Osselin,
Anthony Ladd



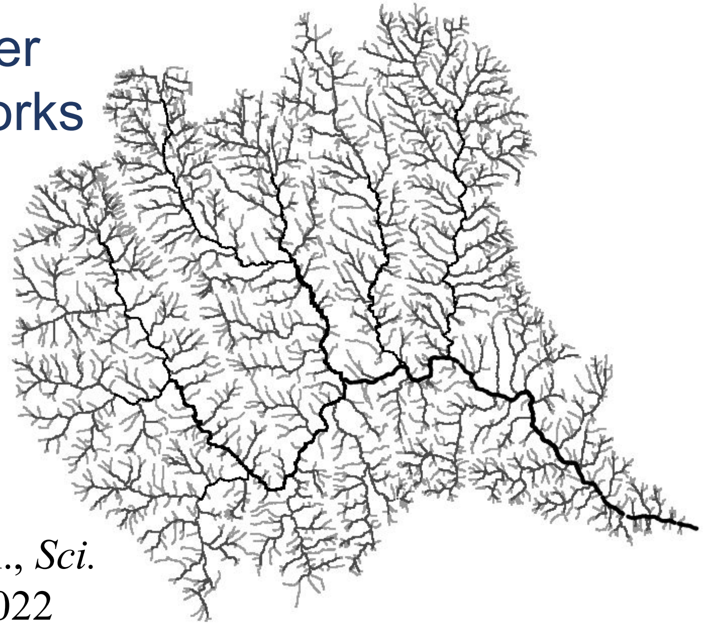
Capillary networks



Rojek & Wrzos et al., *APL Bioeng.*, 2024

Spatial transport networks

River networks



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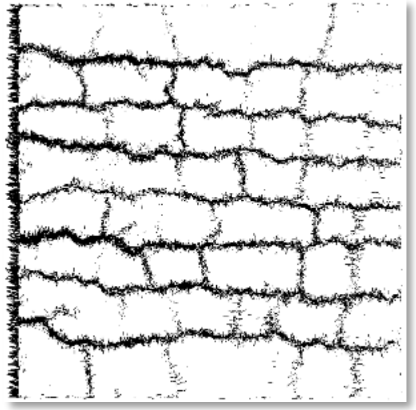

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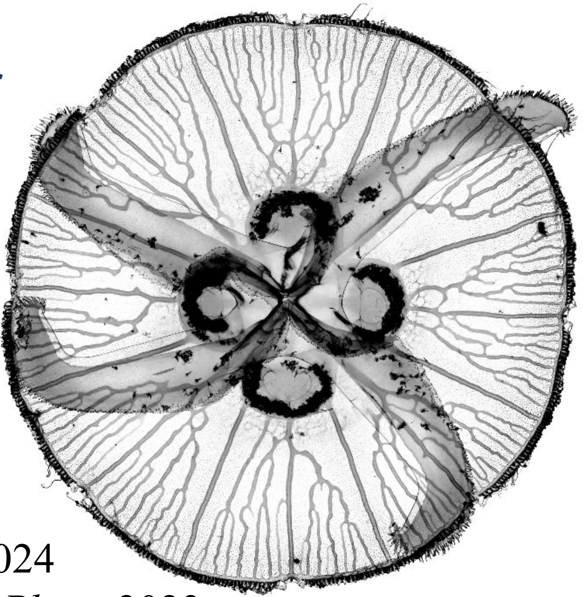

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Microfluidic experiments: fracture dissolution

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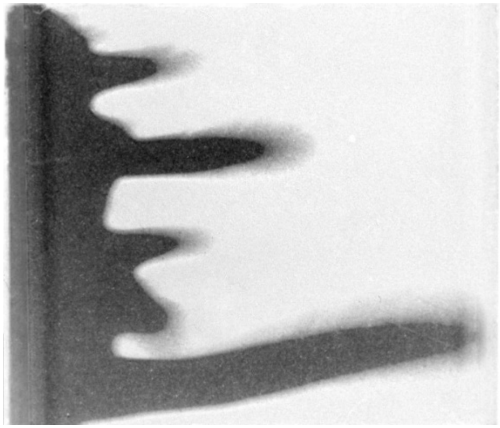
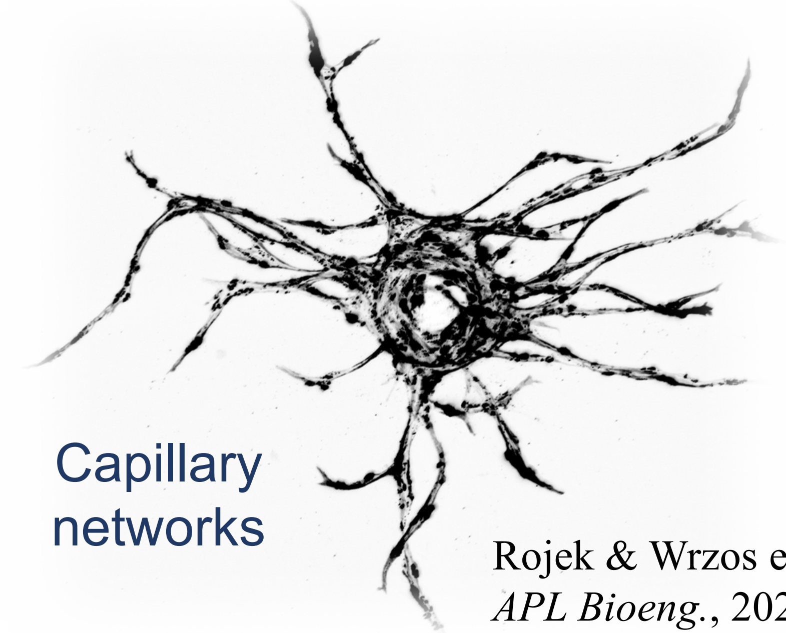




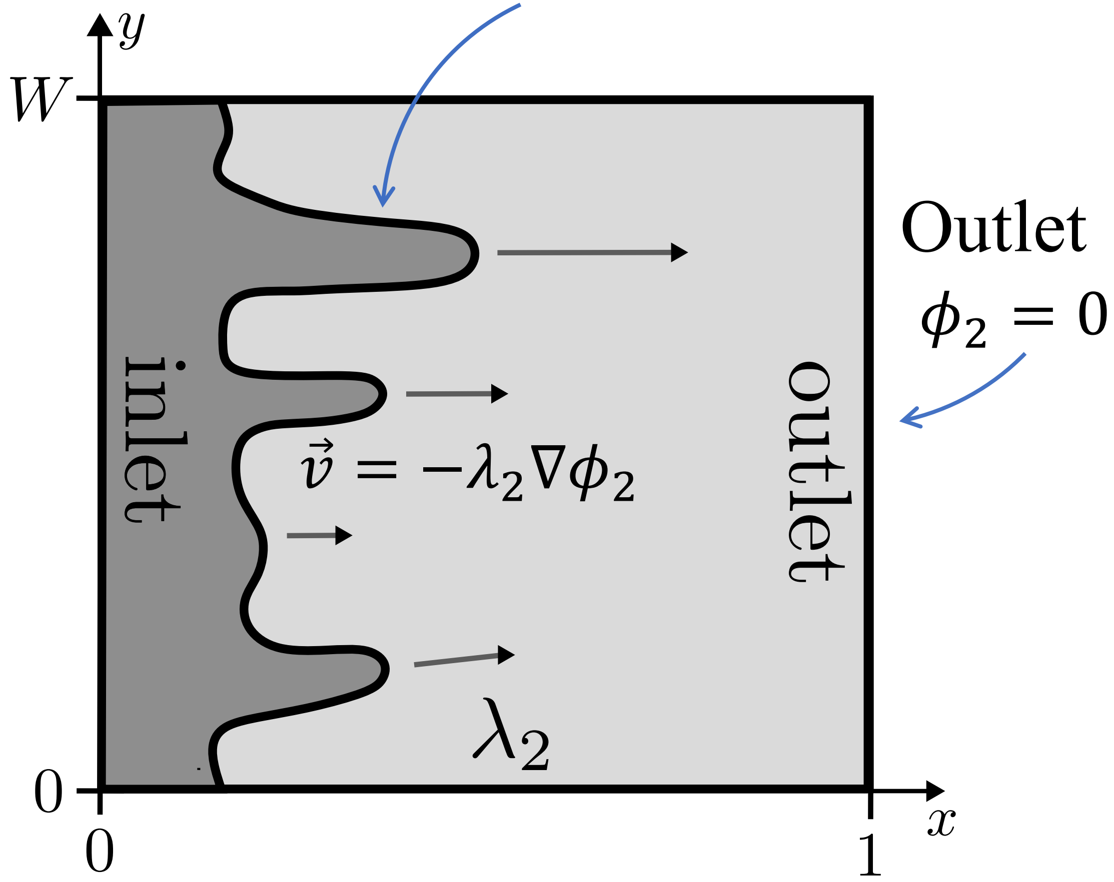
Capillary networks

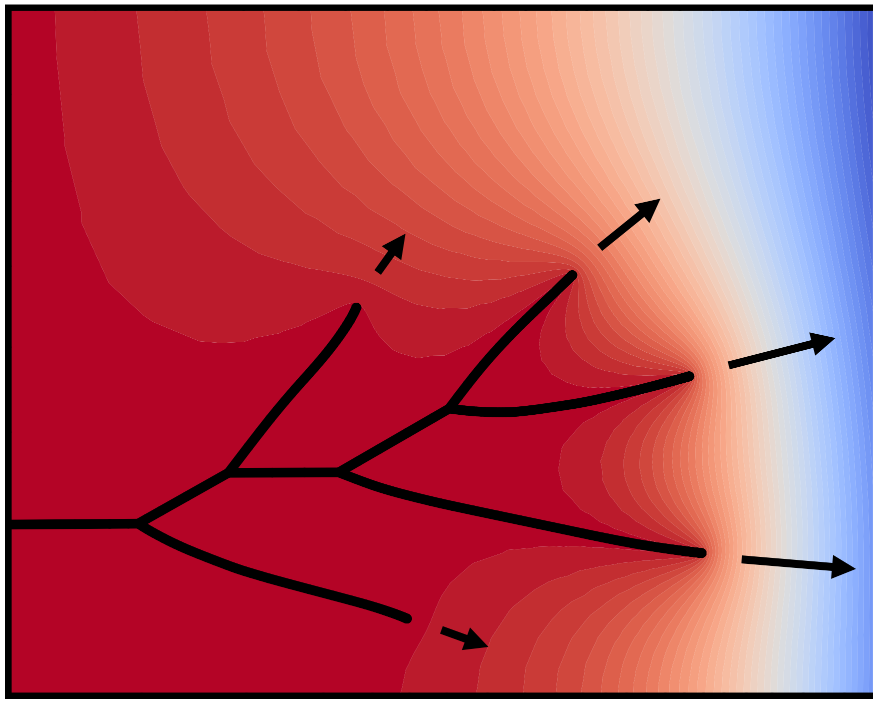
Rojek & Wrzos et al., *APL Bioeng.*, 2024

Laplacian growth

Laplace equation in the domain: $\Delta\phi_2(\vec{x}) = 0$

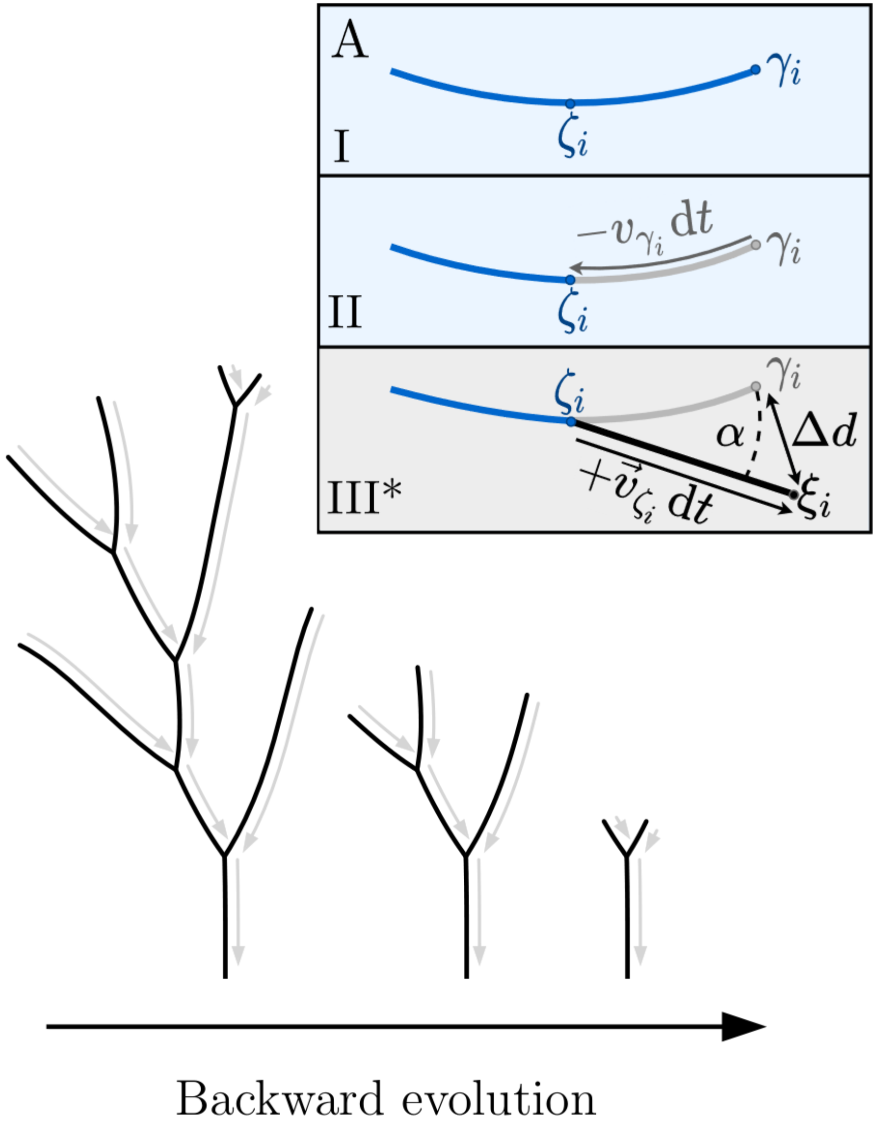
Interface $\phi_2 = 1$ + regularization or… **thin finger model**

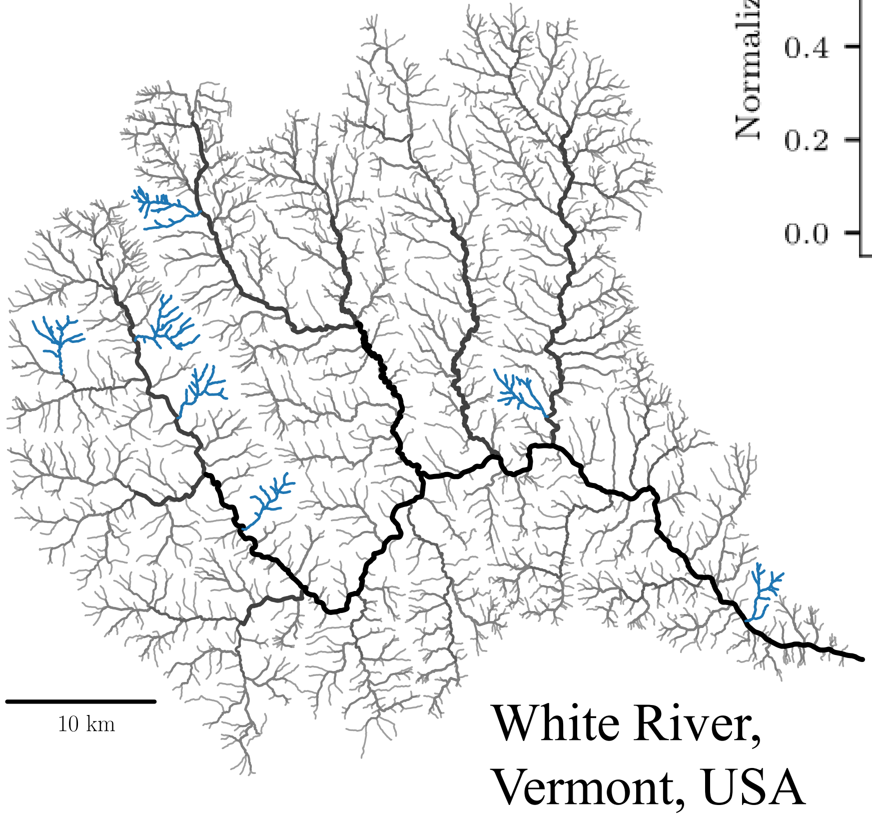


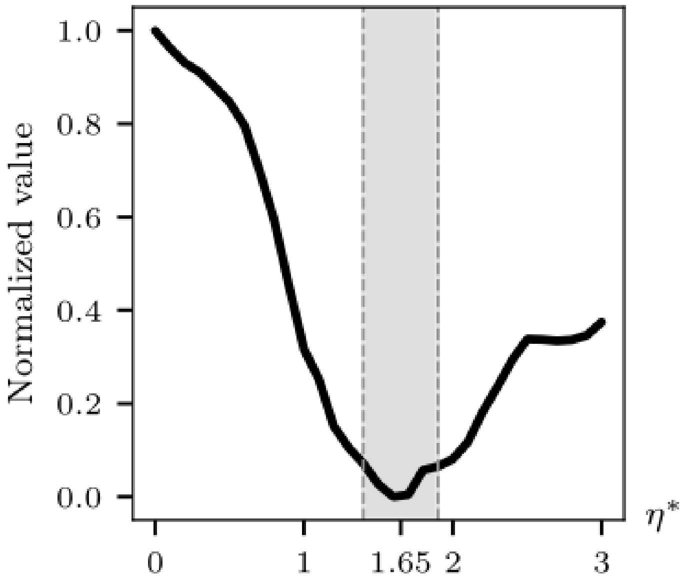


The Laplace equation solved with Finite Elements Method

Backward evolution







SŻ, P. Morawiecki, H. J. Seybold, and P. Szymczak, 'Through history to growth dynamics: deciphering the evolution of spatial networks', *Sci Rep*, doi: [10.1038/s41598-022-24656-x](https://doi.org/10.1038/s41598-022-24656-x).

Laplacian growth

(with drop of potential inside the invading phase)

Laplace equation:

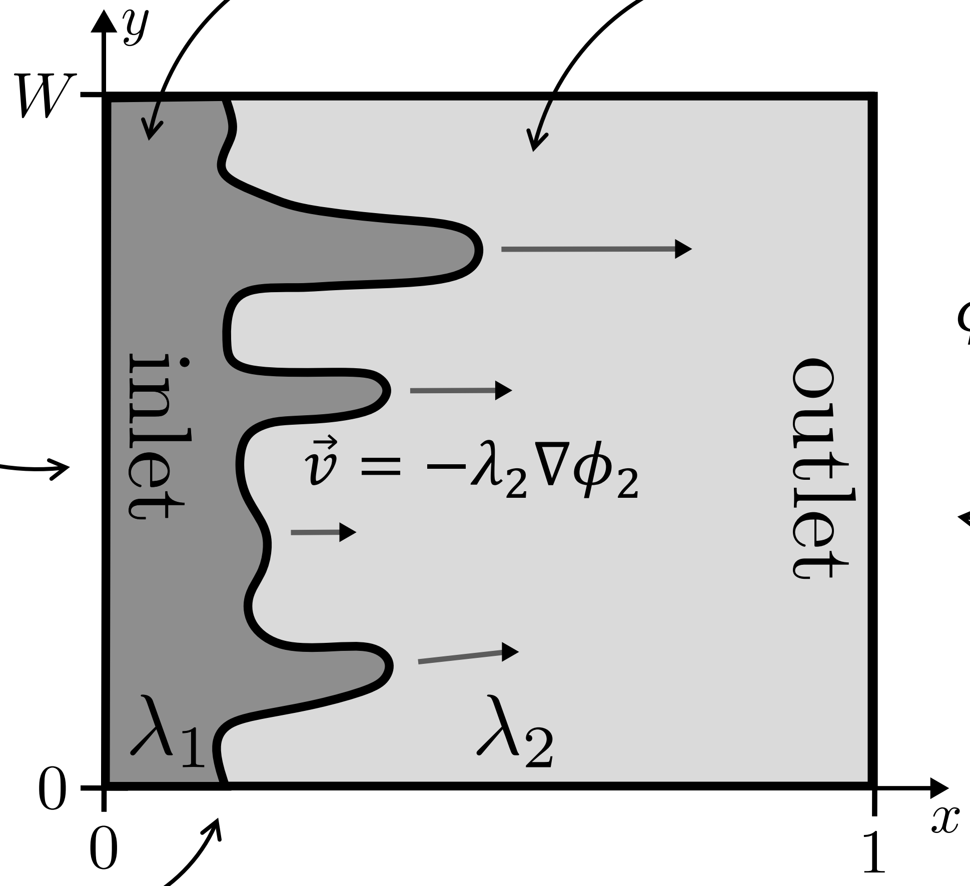
$\Delta\phi_1(\vec{x}) = 0$ $\quad\quad$ $\Delta\phi_2(\vec{x}) = 0$

Inlet:

$\phi_1(x = 0) = 1$

Outlet:

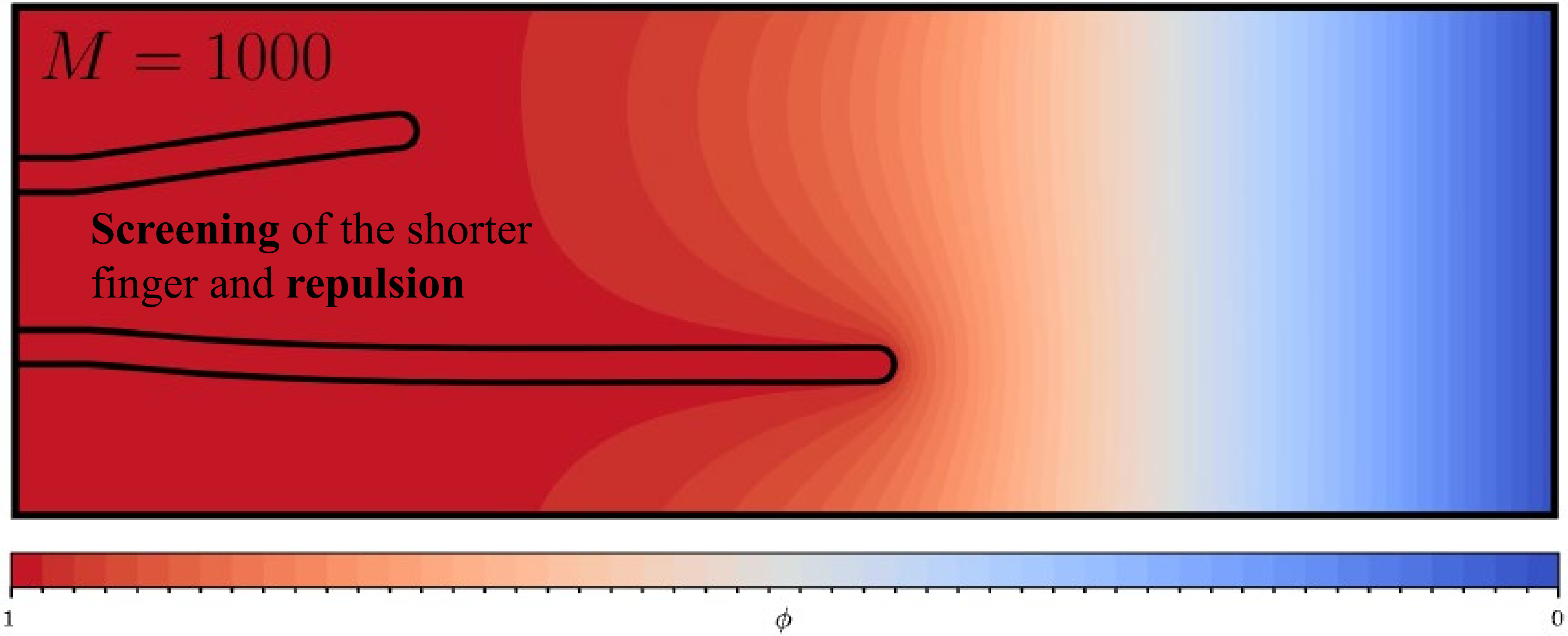
$\phi_2(x = 1) = 0$



Interface: $\quad \phi_1(\vec{x}) = \phi_2(\vec{x})$

$\lambda_1(\nabla\phi_1(\vec{x}))_{\mathrm{n}} = \lambda_2(\nabla\phi_2(\vec{x}))_{\mathrm{n}}$

Dynamical loop formation



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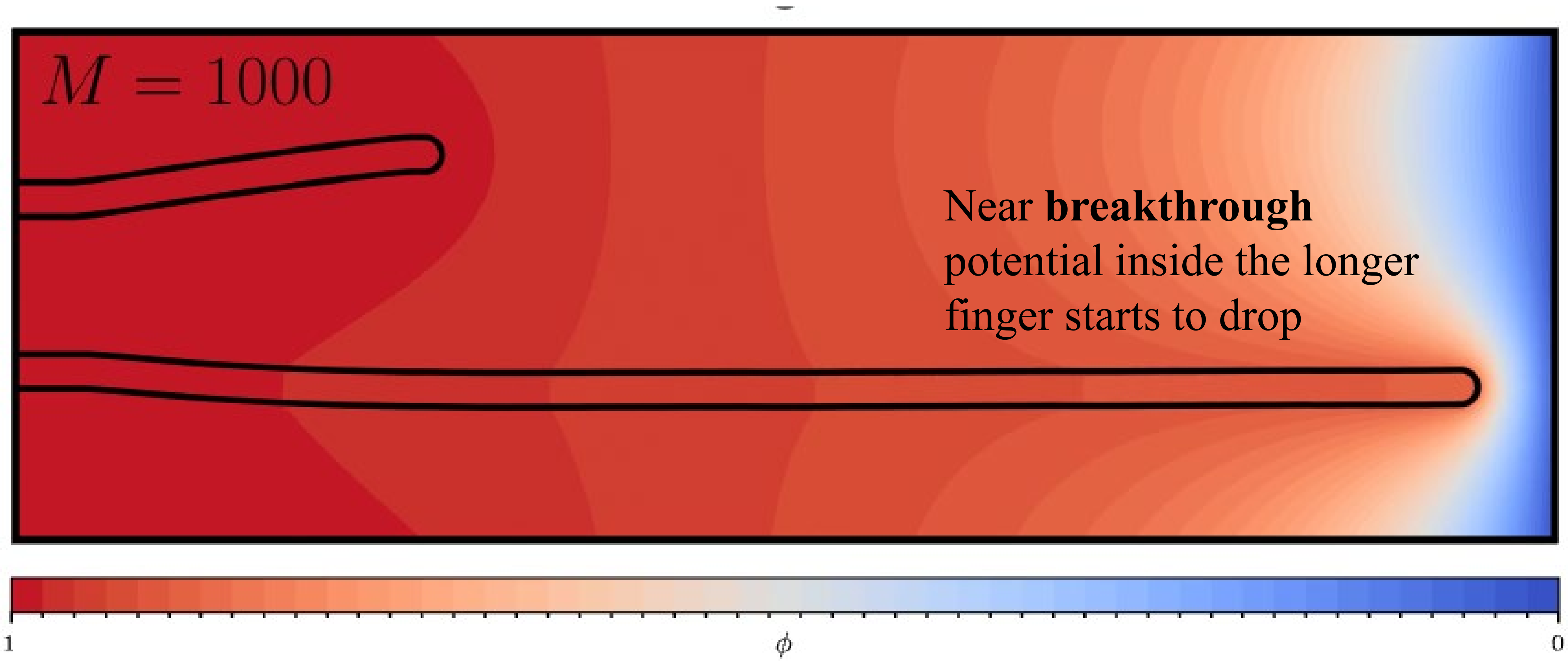

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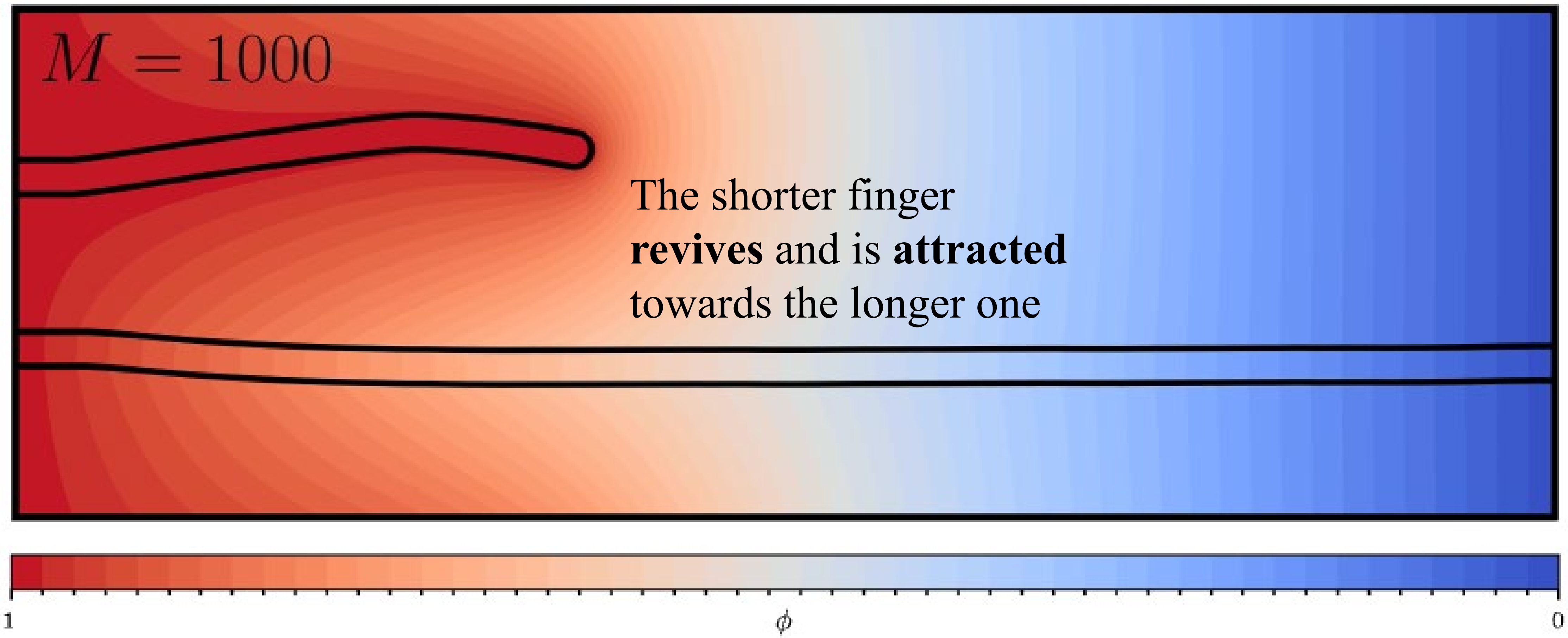

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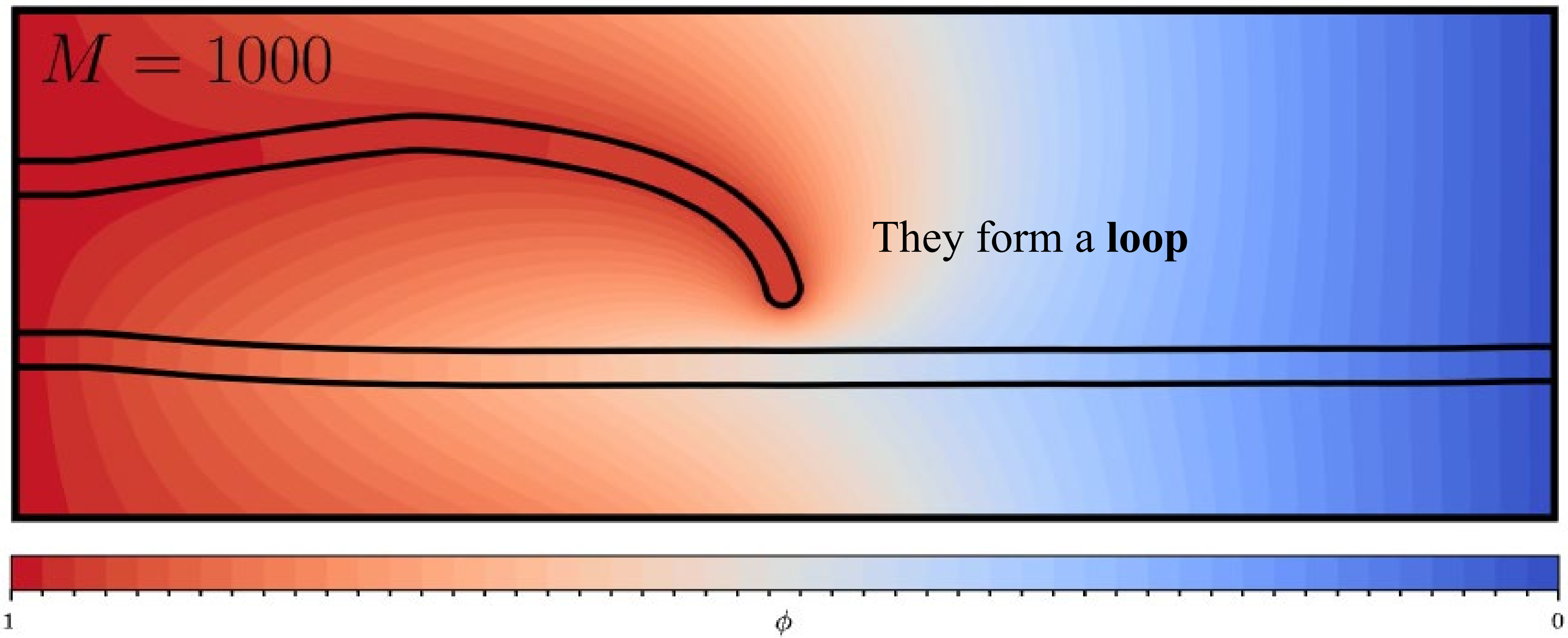
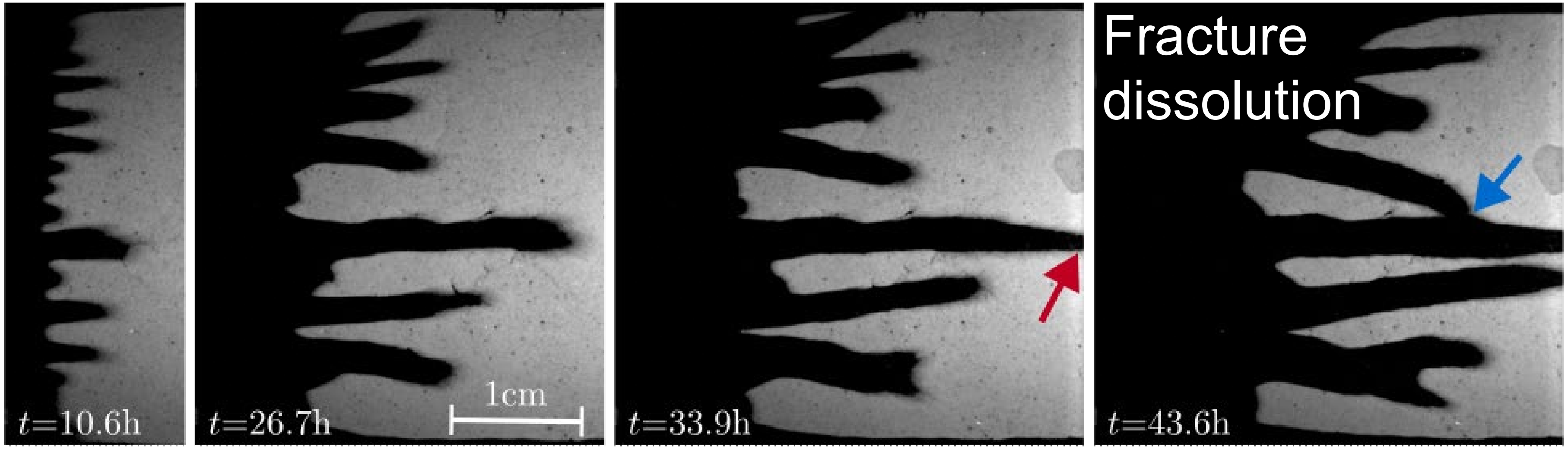


SŻ, A. J. M. Cornelissen, F. Osselin, S. Douady, and P. Szymczak, ‘Breakthrough-induced loop formation in evolving transport networks’, *PNAS*, doi: 10.1073/pnas.2401200121

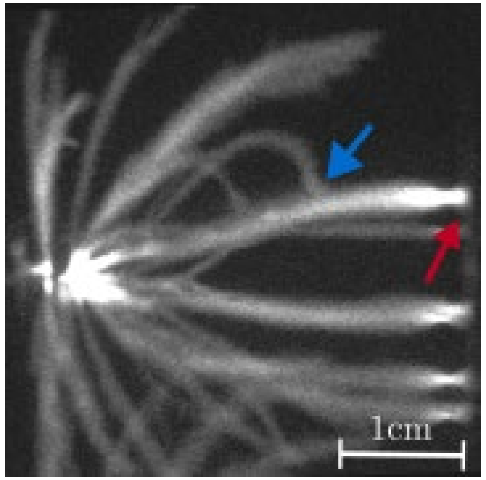
Breakthrough-Induced Loop Formation

Breakthrough

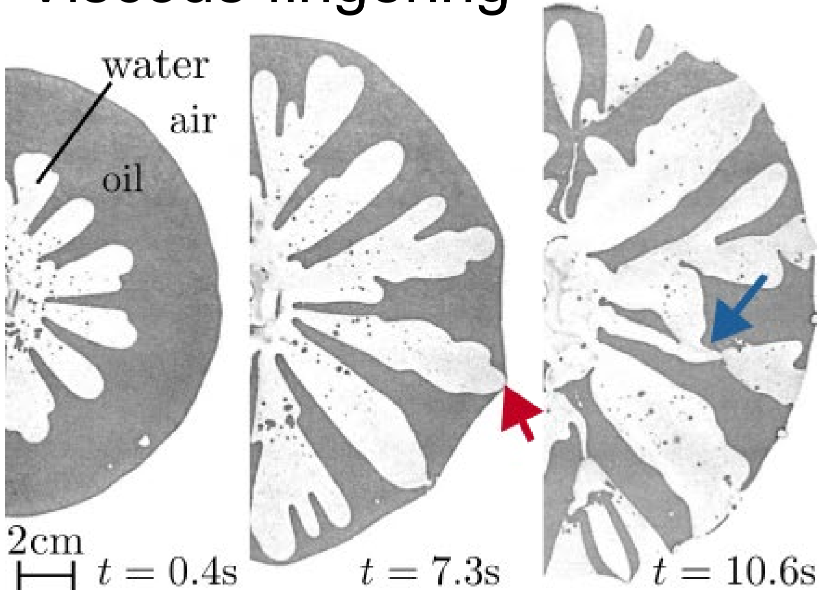
Reconnection



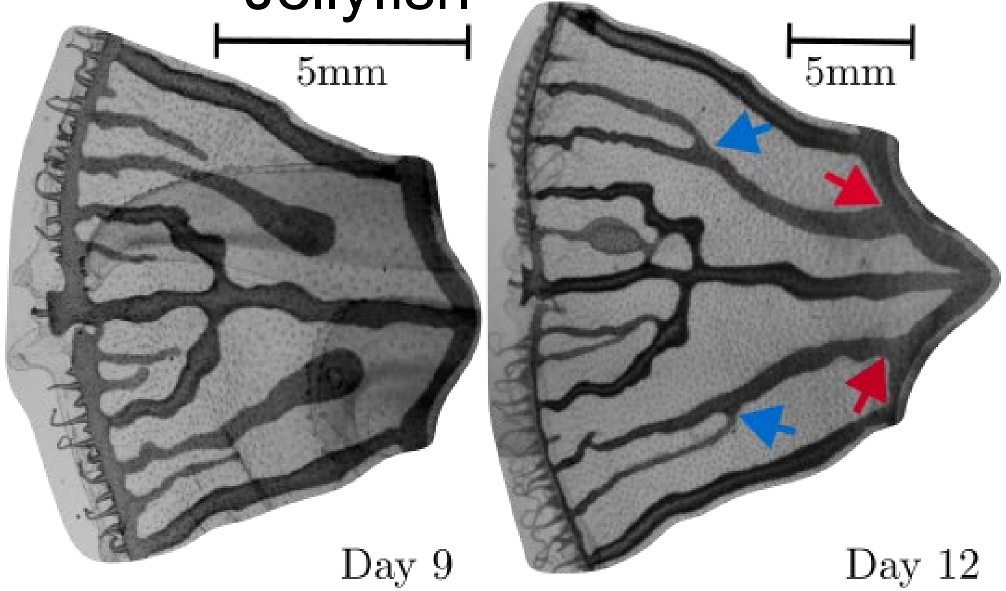
Streamer discharges



Viscous fingering



Jellyfish



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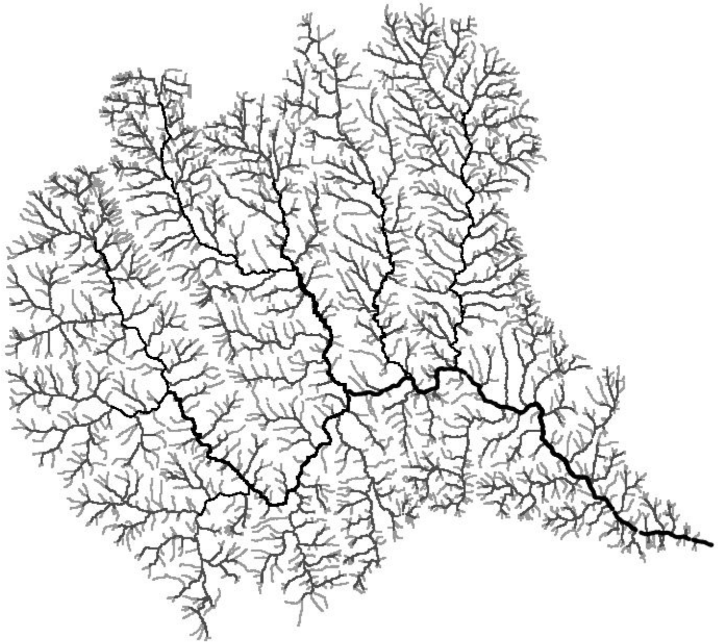
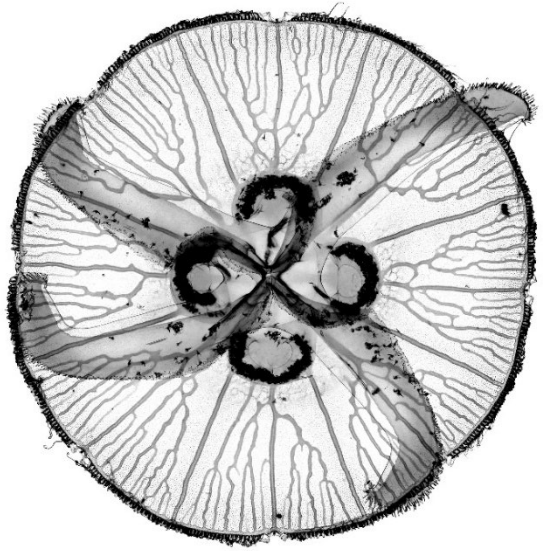
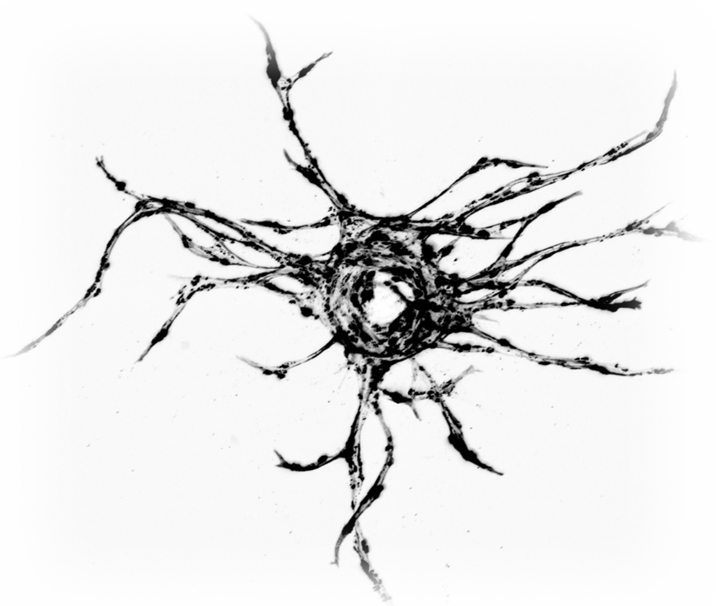

Thank you!